Information fusion and conflict management

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Outline

1. Introduction to information fusion
2. Theory of belief functions for information fusion
3. Managing conflict
4. Decisions with conflicting bbas
What is information?

: The traffic is dense on the runway, ready for takeoff.
What is information?

: The traffic is dense on the runway, ready for takeoff.

Imprecise proposition
What is information?

: The traffic is dense on the runway, ready for takeoff.

Imprecise proposition

: Ten aircrafts were on the runway, ready for takeoff.
What is information?

: The traffic is dense on the runway, ready for takeoff.

Imprecise proposition

: Ten aircrafts were on the runway, ready for takeoff.

Precise proposition
What is information?

: The traffic is dense on the runway, ready for takeoff.

Imprecise proposition

: Ten aircrafts were on the runway, ready for takeoff.

Precise proposition

Imprecision is a kind of imperfection of information
What is information?

: A boeing 777 is at the position $x, y, z$
What is information?

: A boeing 777 is at the position $x, y, z$

Bad weather
Uncertain proposition
What is information?

: A boeing 777 is at the position $x, y, z$

Bad weather
Uncertain proposition

Good weather
Certain proposition
What is information?

: A boeing 777 is at the position $x$, $y$, $z$

**Bad weather**
Uncertain proposition

**Good weather**
Certain proposition

Uncertainty is another kind of imperfection of information
How are the sources?

A: I am sure that is a fighter plane.

B: That is maybe a A380

**FUSION**
How are the sources?

A: I am sure that is a fighter plane.

B: That is maybe a A380

**Conflict** of information sources has to be solved

**FUSION**
How are the sources?

A: I am sure that is a fighter plane.
B: That is maybe a A380

FUSION

A is often mistaken
B is never wrong
How are the sources?

: I am sure that is a fighter plane.

A

: That is maybe a A380

B

FUSION

: That is maybe a A380

A is often mistaken

B is never wrong
How are the sources?

: I am sure that is a fighter plane.

A

: That is maybe a A380

B

**FUSION**

**Reliability** of information sources has to be considered

A is often mistaken

B is never wrong
Assume all the sources are completely reliable giving totally certain and precise information
Assume all the sources are completely reliable giving totally certain and precise information.
Assume all the sources are completely reliable giving totally certain and precise information.

: This is an A380

: This is an A380

: This is an A380

FUSION
Imperfections on information

- Uncertainty: Degree of conformity to the reality
- Imprecision: quantitative default of knowledge on the information contain
- Incompleteness: lack of information

Sources of information

- Reliability: to give an indisputable information
- Independence: Sources are not linked (assumption often made)
- Conflicting sources: Sources give information in contradiction
Information fusion

**Goal:** To combine information coming from many imperfect sources in order to improve the decision making taking into account of imprecisions and uncertainties.

To combine information coming from imperfect sources leads fatally to conflict.

Three actions are possible face to imperfections:

1. We can try to suppress its
2. We can tolerate its and so we need robust algorithms against these imperfections
3. We can model its

To model imperfections: uncertainty theories:
- Probability theory (Bayesian approach) or possibility theory or the theory of belief functions
Fusion architecture for classifiers fusion

$s$ sources $S_1, S_2, \ldots, S_s$ that must take a decision on an observation $x$ in a set of $n$ classes $x \in \Omega = \{\omega_1, \omega_2, \ldots, \omega_n\}$ classes

\[
S_1 \begin{bmatrix}
\omega_1 & \ldots & \omega_i & \ldots & \omega_n \\
M_1^1(x) & \ldots & M_i^1(x) & \ldots & M_n^1(x) \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
M_1^j(x) & \ldots & M_i^j(x) & \ldots & M_n^j(x) \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
M_1^s(x) & \ldots & M_i^s(x) & \ldots & M_n^s(x)
\end{bmatrix}
\]

(1)
Steps of information fusion

Problems

- How to combine the information coming from all the sources?
- How to take the decision?

Under-problems depending on the application

- How to model information? i.e. choice of a formalism
- How to estimate the model parameters?

4 steps

1. Modeling
2. Estimation
3. Combination
4. Decision
Voting process

**Modeling:** Indicator functions

\[ M_i^j(x) = \begin{cases} 1 & \text{if } S_j : x \in \omega_i \\ 0 & \text{otherwise} \end{cases} \]

**Estimation:** \( \alpha_{ij} \): reliability of a source for a given class

**Combination:**

\[ M_i^E(x) = \sum_{j=1}^{s} \alpha_{ij} M_i^j(x) \]

**Decision:**

- \( x \in \omega_k \) if \( M_k^E(x) = \max_i M_i^E(x) \geq c.s + b(x) \)
- \( x \in \omega_{n+1} \) otherwise i.e. no decision
- \( c \in [0, 1] \), \( b(x) \) function of \( M_k^E(x) \)
Voting process: A result

▶ Assumptions
  ▶ Statistical independence of sources
  ▶ Same probability of success \( p \)
  ▶ \( s \) odd and \( > 2 \) (same kind of result if \( s \) is even)

▶ Then
With \( P_R \) the probability of success after fusion by voting process
  ▶ If \( p > 0.5 \), \( P_R \) tends toward 1 with \( s \)
  ▶ If \( p < 0.5 \), \( P_R \) tends toward 0 with \( s \)
  ▶ If \( p = 0.5 \), \( P_R = 0.5 \) for all \( s \)

Behind this result: The majority should have right for fusion.
**Modeling:** A probability is a positive and additive measure, $p$ is defined on a $\sigma$-algebra of $\Omega = \{\omega_1, \omega_2, \ldots, \omega_n\}$ and takes values in $[0,1]$. It verifies: $p(\emptyset) = 0$, $p(\Omega) = 1$, $\sum_{X \in \Omega} p(X) = 1$

**Estimation:** Choice of the distribution, and/or estimation of parameters

**Combination:** Bayes rule

$$p(x \in \omega_i / S_1, \ldots, S_s) = \frac{p(S_1, \ldots, S_s / x \in \omega_i)p(x \in \omega_i)}{p(S_1, \ldots, S_s)} \quad (2)$$

Independence assumption must of the time necessary

**Decision:** *a posteriori* maximum, likelihood maximum, mean maximum, *etc.*
Limits of the theory of probabilities

- Difficulties to model the absence of knowledge (ex: Sirius)
- Constraint on the classes (exhaustive and exclusive)
- Constraint on the measures (additivity)

Example of Smets on the matter of additivity

If one symptom \( f \) (for fiver) is always true when a patient get a illness \( A \) (flu) \( (p(f \mid A) = 1) \), and if we observe this symptom \( f \), then the probability of the patient having \( A \) increases (because \( p(A \mid f) = p(A)/p(f) \) so \( p(A \mid f) \geq p(A) \)).

The additivity constraint require then that the probability of the patient having not \( A \) decreases:

\[ p(A \mid f) = 1 - p(A \mid f) \text{ so } p(A \mid f) \leq p(A) \]

While there is no reason if the symptom \( f \) can be also observe in some other diseases.
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Bases on Belief functions

- Use of functions defined on sub-sets instead of singletons such as probabilities
- Discernment frame: $\Omega = \{\omega_1, \ldots, \omega_n\}$, with $\omega_i$ are exclusive and exhaustive classes
- Power set: $2^\Omega = \{\emptyset, \{\omega_1\}, \{\omega_2\}, \{\omega_1 \cup \omega_2\}, \ldots, \Omega\}$.
- Several functions in one to one correspondence model uncertainty and imprecision: mass functions, belief functions, plausibility functions
- Extension of $2^\Omega$ to $D^\Omega$, hyper power set in order to model the conflicts
  - $D^\Omega$ closed set by union and intersection operators
  - $D^\Omega_r$: reduced set with constraints $(\omega_2 \cap \omega_3 \equiv \emptyset)$
The basic belief functions (bba or mass functions) are defined on $2^\Omega$ and take values in $[0, 1]$

Normalization condition: $\sum_{X \in 2^\Omega} m(X) = 1$

A focal element is an element $X$ of $2^\Omega$ such as $m(X) > 0$

Closed world: $m(\emptyset) = 0$

We note $m_j$ the mass function of the source $S_j$
Mass functions

Special cases:

- If only focal elements are $\omega_i$ then $m_j$ is a probability
- $m_j(\Omega) = 1$: total ignorance of $S_j$
- Categorical mass function: $m_j(X) = 1$ (noted $m_X$): $S_j$ has an imprecise knowledge
- $m_j(\omega_i) = 1$: $S_j$ has a precise knowledge
- Simple mass functions $X^w$:
  $m_j(X) = w$ and $m_j(\Omega) = 1 - w$: $S_j$ has an uncertain and imprecise knowledge
From (Shafer, 1976):
\[ m_j^\alpha(X) = \alpha_j m_j(X), \forall X \in 2\Omega \]
\[ m_j^\alpha(\Omega) = 1 - \alpha_j (1 - m_j(\Omega)) \]

\( \alpha_j \in [0, 1] \) discounting coefficient can be seen as the reliability of the source \( S_j \).
If \( \alpha_j = 0 \) the source are completely unreliable, all the mass is transferred on \( \Omega \), the total ignorance.

The discounting process increases the intervals \([\text{bel}_j, \text{pl}_j]\) (and so reduces the global conflict in the conjunctive combination rule).
s sources $S_1, S_2, \ldots, S_s$ that must take a decision on an observation $x$ in a set of $n$ classes $x \in \Omega = \{\omega_1, \omega_2, \ldots, \omega_n\}$ classes.

\[
\begin{bmatrix}
\omega_1 & \cdots & \omega_i & \cdots & \omega_n \\
S_1 & m_1^1(x) & \cdots & m_i^1(x) & \cdots & m_n^1(x) \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
S_j & m_j^1(x) & \cdots & m_i^j(x) & \cdots & m_n^j(x) \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
S_s & m_s^1(x) & \cdots & m_i^s(x) & \cdots & m_n^s(x)
\end{bmatrix}
\]
Distance based model (Denœux 1995)

Only $\omega_i$ and $\Omega$ are focal elements, $n \times s$ sources (experts)

- Prototypes case ($x_i$ center of $\omega_i$). For the observation $x$

\[
m^i_j(\omega_i) = \alpha_{ij} \exp[-\gamma_{ij} d^2(x, x_i)]
\]
\[
m^i_j(\Omega) = 1 - \alpha_{ij} \exp[-\gamma_{ij} d^2(x, x_i)]
\]

- $0 \leq \alpha_{ij} \leq 1$: discounting coefficient and $\gamma_{ij} > 0$, are parameters to play on the quantity of ignorance and on the form of the mass functions

- The distance allows to give a mass to $x$ higher according to the proximity to $\omega_i$

- belief $k$-nn: we consider the $k$-nearest neighbors instead to $x_i$

- Then we combine the bbas
2 models proposed by Appriou according to three axioms:

1. The \( n \times s \) couples \([m^j_i, \alpha_{ij}]\) are distinct information sources where focal elements are: \( \omega_i, \overline{\omega_i} \) and \( \Omega \).

2. If \( m^j_i(\omega_i) = 0 \) and the information is valid \((\alpha_{ij} = 1)\) then it is certain that \( \omega_i \) is not true.

3. Conformity to the Bayesian approach (case where \( p(S_j|\omega_j) \) is exactly the reality \((\alpha_{ij} = 1)\) for all \( i, j \)) and all the \textit{a priori} probabilities \( p(\omega_i) \) are known.

Need to estimate \( p(S_j|\omega_i) \).
Probabilistic based model

Model 1:

\[
\begin{align*}
    m_j^i(\omega_i) &= \frac{\alpha_{ij} R_j p(S_j|\omega_j)}{1 + R_j p(S_j|\omega_j)} \\
    m_j^i(\overline{\omega_i}) &= \frac{\alpha_{ij}}{1 + R_j p(S_j|\omega_j)} \\
    m_j^i(\Omega) &= 1 - \alpha_{ij}
\end{align*}
\]

with \( R_j \geq 0 \) a normalization factor.

Model 2:

\[
\begin{align*}
    m_j^i(\omega_i) &= 0 \\
    m_j^i(\overline{\omega_i}) &= \alpha_{ij} (1 - R_j p(S_j|\omega_j)) \\
    m_j^i(\Omega) &= 1 - \alpha_{ij} (1 - R_j p(S_j|\omega_j))
\end{align*}
\]

with \( R_j \in [0, (\max_{S_j,i} p(S_j|\omega_j))^{-1}] \)
Probabilistic vs Distance

Difficulties:
- Appriou: learning the probabilities $p(S_j | \omega_j)$
- Denœux: choice of the distance $d(x, x_i)$

Easiness:
- $p(S_j | \omega_j)$ easier to estimate on decisions with the confusion matrix of the classifiers
- $d(x, x_i)$ easier to choose on the numeric outputs of classifiers (ex.: Euclidean distance)
Assume: two cognitively independent and reliable sources $S_1$ and $S_2$.

The conjunctive rule is given for $m_1$ and $m_2$ bbas of $S_1$ and $S_2$, for all $X \in 2^\Omega$, with $X \neq \emptyset$ by:

$$m_{\text{Conj}}(X) = \sum_{Y_1 \cap Y_2 = X} m_1(Y_1)m_2(Y_2) \quad (3)$$

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<tr>
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<th>$\emptyset$</th>
<th>$\omega_1$</th>
<th>$\omega_2$</th>
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<td>0.03</td>
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</table>
Dempster’s rule:

\[ m_D(X) = \frac{1}{1 - \kappa} m_{\text{conj}}(X) \]  \hspace{1cm} (4)

where \( \kappa = \sum_{A \cap B = \emptyset} m_1(A)m_2(B) \) is generally called conflict or global conflict. That is the sum of the partial conflicts.

That is not a conflict measure (Liu, 2006).

Conjunctive rules are not idempotent.
### Idempotence

Information fusion and conflict management, A. Martin

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<tr>
<td>$m_{Conj}$</td>
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The mass on the empty set obtained by the combination of two identical bbad is not null.
In general the decision is made on $\Omega$ and not on $2^\Omega$

- Pessimist: $\max_{\omega \in \Omega} bel(\omega)$
- Optimist: $\max_{\omega \in \Omega} pl(\omega)$
- Compromise: $\max_{\omega \in \Omega} betP(\omega)$

Pignistic probability:

$$betP(\omega) = \sum_{Y \in 2^\Omega, \omega \cap Y \neq \emptyset} \frac{1}{|Y|} \frac{m(Y)}{1 - m(\emptyset)}$$  (5)
Decision on the unions

- Decision on \( 2^\Omega \)
- The decision functions \( f_d \) (belief, plausibility, pignistic probability, etc.) increase by inclusion (Appriou, 2014)

\[
A = \arg\max_{X \in 2^\Omega} (m_d(X) f_d(X)),
\]

where

\[
m_d(X) = \left( \frac{K_d \lambda X}{|X| r} \right)
\]

with \( r \in [0, 1] \), is a weighted factor of the wanted precision of the decision:
- \( r = 1 \): singleton,
- \( r = 0 \): ignorance
1. Introduction to information fusion
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Definition  The conflict in the theory of belief functions can be defined by the contradiction between two or more mass functions.

► The sources are not reliable: reliability and conflict are very linked in this case

► The discernment frame is not exhaustive: the closed world assumption must be avoid

► The sources do not express on the same phenomena: bbas must not be combined

Let note $\text{Conf}(m_1, m_2)$ a conflict measure between two mass functions $m_1$ and $m_2$. 
Many measures are called conflict in the theory of belief functions but are not conflict.

The global conflict contains an indication of the conflict between bbas, but not only.

Some terms such as internal conflict, discord, contradiction are not conflict.

Some uncertainty measures such as entropic measures can be sometimes called conflict, but are not conflict.
Auto-conflict: a discord measure

Introduced by Osswald and Martin, 2006, the auto-conflict of order \( s \) for one expert is given by:

\[
a_s = (\bigcap_{j=1}^{s} m) (\emptyset).
\]

(6)

where \( \bigcap \) is the conjunctive operator.

The following property holds:

\[
\lim_{s \to +\infty} a_s = 1 \quad \text{if the intersection of all focal elements of } m \text{ is empty}
\]

The global conflict must be weighted with the auto-conflict.

Another discord measure is given by the internal conflict of Schubert (2012).
Based on the global conflict coming from the conjunctive rule:

\[
\text{Conf}(m_1, m_2) = m_{\text{Conj}}(\emptyset) \quad \text{(point of view of Smets (2007))}
\]

\[
\text{Conf}(m_1, m_2) = -\ln(1 - m_{\text{Conj}}(\emptyset)) \quad \text{Yager (1983)}
\]

- **pros:** \(\text{Conf}(m_1, m_\Omega) = 0\)
  where \(m_\Omega(\Omega) = 1\) is the total ignorance

- **cons:**
  1. \(\text{Conf}(m_1, m_1) \neq 0\) because of the non-idempotence
  2. \(m_{\text{Conj}}(\emptyset)\) contains a part of the auto-conflict of \(m_1\) and \(m_2\)
  3. The conjunctive rule needs the independence assumption between the sources: There is no reason to get a link between independence and conflict
Conflict measure: based on distance

Conf\((m_1, m_2) = d(m_1, m_2)\) (Martin et al., 2008)

Conflict between an expert \(S_j\) and the \(s-1\) other experts is given by the the mean of conflicts two by two:

\[
\text{Conf}(m_j, m_\mathcal{E}) = \frac{1}{s-1} \sum_{e=1, e \neq j}^{s} \text{Conf}(j, e) \tag{7}
\]

Another definition is given by: \(\text{Conf}(m_j, m_s) = d(m_j, m_\mathcal{E})\)

where \(m_\mathcal{E}\) is the bba of the artificial expert build by the combined bbas of the \(s-1\) other experts of \(\mathcal{E}\) without the expert \(S_j\).

- **pros:** \(\text{Conf}(m_1, m_1) = 0\)
- **cons:** \(\text{Conf}(m_1, m_\Omega) \neq 0\)
The Jousselme et al. (2001) distance can be done by:

\[
d(m_1, m_2) = \sqrt{\frac{1}{2} (m_1 - m_2)^T D(m_1 - m_2)},
\]

where \(D\) is an \(2^{|\Theta|} \times 2^{|\Theta|}\) matrix based on Jaccard dissimilarity

\[
D(A, B) = \begin{cases} 
1, \text{ if } A = B = \emptyset, \\
\frac{|A \cap B|}{|A \cup B|}, \forall A, B \in 2^\Theta.
\end{cases}
\]

Distance based on plausibility (Jousselme and Maupin (2011)):

\[
d(m_1, m_2) = 1 - \frac{\text{pl}_1^T \cdot \text{pl}_2}{\|\text{pl}_1\| \|\text{pl}_2\|},\text{ where } \text{pl} \text{ is the plausibility function and } \text{pl}_1^T \cdot \text{pl}_2 \text{ the vector product in } 2^n \text{ space of both plausibility functions.}
\]
We define a conflict measure between two mass functions $m_1$ and $m_2$ by (Martin, 2012):

$$\text{Conf}(m_1, m_2) = (1 - \delta_{\text{inc}}(m_1, m_2))d(m_1, m_2)$$  \hspace{1cm} (8)$$

where $d$ is the distance, $\delta_{\text{inc}}$ is a degree of inclusion. This measure holds axioms:

1. **Non-negativity**: $\text{Conf}(m_1, m_2) \geq 0$
2. **Identity**: $\text{Conf}(m_1, m_1) = 0$
3. **Symmetry**: $\text{Conf}(m_1, m_2) = \text{Conf}(m_2, m_1)$
4. **Normalization**: $0 \leq \text{Conf}(m_1, m_2) \leq 1$
5. **Inclusion**: $\text{Conf}(m_1, m_2) = 0$, iff $m_1 \subseteq m_2$ or $m_2 \subseteq m_1$

The mass functions cannot be in conflict if one is included in the other one.
If the conflict comes from unreliable sources: reliability estimation

**Assumption**: a source is unreliable if it is in conflict with the other sources

**Reliability measure**: such as a decreasing function of the conflict measure

\[
\alpha_j = f(\text{Conf}(j, s))
\]
\[
\alpha_j = (1 - \text{Conf}(j, s)^\lambda)^{1/\lambda}
\]

Integration of the reliability measure by discounting

\[
m_j^\alpha(X) = \alpha_j m_j(X), \quad \forall X \in 2^\Omega
\]
\[
m_j^\alpha(\Omega) = 1 - \alpha_j (1 - m_j(\Omega))
\]
Managing the conflict in the combination

Zadeh example

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<tr>
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Solutions

- Conflict coming from a false assumption of closed world
- Conflict coming from the assumption of source’s independence
- Conflict coming from source’s ignorance assumption
- Conflict coming from source reliability assumption
- Conflict coming from a number of sources
False assumption of closed world

- Closed world: Frame of discernment assumed to be exhaustive
- Smets interpreted $m(\emptyset) > 0$ such as another element and used the conjunctive rule $m_{\text{Conj}}$
- $m(\emptyset)$ is composed of all the partial conflict: we can considered such as many unknown elements.
The PCR6 (Martin et Osswald, 2006, 2007)

This rule transfers the partial conflicts on the elements that generate it, proportionally to their mass functions.

\[
m_{\text{PCR6}}(X) = m_{\text{Conj}}(X) + \sum_{\substack{Y \in D^\Omega, \\
X \cap Y \equiv \emptyset}} \left( \frac{m_1(X)^2 m_2(Y)}{m_1(X) + m_2(Y)} + \frac{m_2(X)^2 m_1(Y)}{m_2(X) + m_1(Y)} \right)
\]

\[
m_{\text{PCR6}}(X) = m_{\text{Conj}}(X) + \sum_{j=1}^{s} m_i(X)^2 \sum_{\substack{Y_{\sigma_j(j')} \cap X = \emptyset \\
Y_{\sigma_j(j')} \cap \bigcap_{j'=1}^{s-1} Y_{\sigma_j(j')} = \emptyset}} \left( \frac{\prod_{j'=1}^{s-1} m_{\sigma_{j}(j')} (Y_{\sigma_j(j')})}{m_j(X) + \sum_{j'=1}^{s-1} m_{\sigma_{j}(j')} (Y_{\sigma_j(j')})} \right)
\]
Assumption of source’s independence

- If dependent sources: combination rule has to be idempotent
- The simplest way: the average of the mass functions:

\[ m_M(X) = \frac{1}{s} \sum_{j=1}^{s} m_j(Y_j). \quad (10) \]

- The cautious rule of \( S \) non-dogmatic mass functions \( m_j, j = 1, 2, \ldots, S \):

\[ m_{\text{cau}}(X) = \bigcap_{A \subset \Omega} A^{\bigwedge_{j=1}^{S} w_j(A)}, \quad (11) \]

where \( A^{w_j(A)} \) is the simple support function focused on \( A \) with weight function \( w_j(A) \) issued from the canonical decomposition of \( m_j \). Note also that \( \bigwedge \) is the min operator.
Source’s ignorance

Yager (1987) rule

- Assumption: the global conflict comes from the ignorance
- Stay in closed world
- The mass of the empty set is transferred on the global ignorance $\Omega$

\[
\begin{align*}
m_Y(X) &= m_{\text{Conj}}(X), \forall X \in 2^\Omega \setminus \{\emptyset, \Omega\} \\
m_Y(\Omega) &= m_{\text{Conj}}(\Omega) + m_{\text{Conj}}(\emptyset) \\
m_Y(\emptyset) &= 0.
\end{align*}
\]
Dubois and Prade (1988) rule

- Assumption: Partial conflict comes from the partial ignorances
- The partial conflict are transfered on the partial ignorances

\[
m_{DP}(X) = \sum_{A \cap B = X} m_1(A)m_2(B) + \sum_{A \cup B = X} m_1(A)m_2(B).
\]

- Precise transfer of the global conflict
- Algorithm complexity higher
Disjunctive rule

- If no knowledge about reliability: at least one source is reliable

\[
m_{\text{Dis}}(X) = \sum_{Y_1 \cup \ldots \cup Y_S = X} \prod_{j=1}^{S} m_j(Y_j). \tag{12}
\]

- Main problem: lost of specificity after combination
Florea (2006) rule

▶ Stay in closed world
▶ Propose a global conflict transfer in a such way more the global conflict is high more the rule has a disjunctive comportment.
▶ The rule is given $\forall X \in 2^\Omega$, $X \neq \emptyset$ by:

$$m_{Flo}(X) = \beta_1(\kappa)m_{Dis}(X) + \beta_2(\kappa)m_{Conj}(X),$$

where $\beta_1$ and $\beta_2$ have $\kappa = \frac{1}{2}$ like a symmetric weight:

$$\beta_1(\kappa) = \frac{\kappa}{1 - \kappa + \kappa^2},$$

$$\beta_2(\kappa) = \frac{1 - \kappa}{1 - \kappa + \kappa^2}.$$

▶ We have seen that $1/2$ cannot be the symmetric value of the global conflict
▶ Other weights are proposed
Reliability assumption

(Martin et Osswald, 2007): **Mixed rule**

- General distribution of partial conflict

\[
m_{\text{Mix}}(X) = \sum_{Y_1 \cup Y_2 = X} \delta_1 m_1(Y_1)m_2(Y_2) + \sum_{Y_1 \cap Y_2 = X} \delta_2 m_1(Y_1)m_2(Y_2) \tag{13}
\]

- Dubois and Prade’s rule

\[
\delta_1(Y_1, Y_2) = 1 - \delta_2(Y_1, Y_2) = 1 - \mathbb{1}_{Y_1 \cap Y_2 = \emptyset}(Y_1, Y_2)
\]

and Florea’s rule can be seen such as a particular case
Mixed rule: taking into account the specificity
The choice of $\delta_1 = 1 - \delta_2$ can be given from a dissimilarity measure such as:

$$\delta^1_1(Y_1, Y_2) = 1 - \frac{|Y_1 \cap Y_2|}{\min(|Y_1|, |Y_2|)}$$  \hspace{1cm} (14)$$
or from the Jaccard’s dissimilarity:

$$\delta^2_1(Y_1, Y_2) = 1 - \frac{|Y_1 \cap Y_2|}{|Y_1 \cup Y_2|}$$  \hspace{1cm} (15)$$

- If $Y_1 \cap Y_2 = \emptyset$: partial conflicts can be interpreted such as partial ignorances
- If $Y_1 \cap Y_2 \notin \{Y_1, Y_2, \emptyset\}$: transfer on $Y_1 \cap Y_2$ and $Y_1 \cup Y_2$ according to $\delta^1_1$ and $\delta^2_1$
LNS rule: (Zhou et al. 2017)

- **Problems:**
  - Many sources: assumption of the reliability of all the sources difficult to consider
  - Disjunctive rule: at least one source reliable but lost of specificity

- **Assumptions of LNS rule**
  - Majority of sources are reliable
  - The larger extent one source is consistent with others, the more reliable the source is
  - Sources are cognitively independent
LNS rule: (Zhou et al. 2017)

1. Cluster the simple BBAs into $c$ groups based on their focal element
2. Combine the BBAs in the same group
3. Reliability-based discounting
4. Global combine the fused BBAs in different groups
5. Output the fused BBA
LNS rule: (Zhou et al. 2017)

For each mass function $m_j$ we consider the set of mass functions $\{A^w_j, A_k \subset \Omega\}$ coming from the canonical decomposition. If group the simple mass functions $A^w_j$ in $c$ clusters (the number of distinct $A_k$) and denote by $s_k$ the number of simple mass functions in the cluster $k$, the proposed rule is given by:

$$m_{LNS} = \bigcap_{k=1,\ldots,c} (A_k)$$

where

$$\alpha_k = \frac{s_k}{\sum_{i=1}^{c} s_i}.$$
What to do to manage the conflict?

- First, if you have any reliable information on the reliability of the sources: discount the mass functions.
- If there is still global conflict: choose an appropriate rule to manage this conflict.
- There is no an optimal rule for any application.
  1. Choose the rule according to the needed properties (reliability, idempotence, independence, complexity, etc.)
  2. Try some rules and choose the best for your application
Outline

1. Introduction to information fusion
2. Theory of belief functions for information fusion
3. Managing conflict
4. Decisions with conflicting bbas
Decisions with conflicting bbas

- Another point of view: do not manage the conflict and keep it until the decision.
- That is the point of view of Smets with the use of the conjunctive rule and the pignistic probability for the decision.
- Other idea: keep all the partial conflicts during the combination rule:
  - Hyper power set $D^\Omega = \{\omega_1 \cap \omega_2, \omega_1, \omega_2, \omega_1 \cup \omega_2\}$, if $\Omega = \{\omega_1, \omega_2\}$
  - $D_r^\Omega$: reduced set with constraints ($\omega_2 \cap \omega_3 \equiv \emptyset$)
In $D^\Omega$ classic decision can be made on $\Omega$ without partial conflict

- Pessimist: $\max_{\omega \in \Omega} bel(\omega)$
- Optimist: $\max_{\omega \in \Omega} pl(\omega)$
- Compromise: $\max_{\omega \in \Omega} betP(\omega)$

Pignistic probability:

$$GPT(X) = \sum_{Y \in D^\Omega_r, Y \neq \emptyset} \frac{C_M(X \cap Y)}{C_M(Y)} m(Y)$$  \hspace{1cm} (18)$$

where $C_M(X)$ is the cardinality $X$ of $D^\Omega_r$
(Martin et Quidu, 2008) decision process:

1. Reject the elements not in the learned classes

\[
\begin{align*}
\text{bel}(\omega_d) &= \max_{1 \leq i \leq n} \text{bel}(\omega_i) \\
\text{bel}(\omega_d) &\geq \text{bel}(\bar{\omega_d})
\end{align*}
\] (19)

2. Decision on \(2^\Omega\) on the non-rejected elements

The decision functions \(f_d\) (belief, plausibility, pignistic probability, etc.) increase by inclusion (Appriou, 2014)

\[
A = \arg\max_{X \in 2^\Omega} (m_d(X)f_d(X)) , \quad m_d(X) = \left( \frac{K_d \lambda X}{|X|^r} \right)
\] (20)

\(r \in [0, 1]\), \(r = 1\): singleton, \(r = 0\): ignorance
Decision in presence of conflict

Example

Data GESMA

Sand
Rock
Rock OR Sand
Rock AND Sand
Wreck or object
(Martin, 2008): decision on $D^\Omega$

- on $D^\Omega_r$
  \[ A = \arg\max_{X \in D^\Omega_r} (m_d(X)f_d(X)) \]
  \[ m_d(X) = \left( \frac{K_d \lambda X}{C_M(X)r} \right) \]

- on $D$ a subset of $D^\Omega$
  \[ A = \arg\max_{X \in D} (m_d(X)f_d(X)) \]

such as the cardinality
\[ D = \{ X \in D^\Omega_r ; \min_C \leq C_M(X) \leq \max_C \} \]
Decision on conflicting bbas

Decision on $2^\Omega$ (Essaid, et al. 2014)

\[
A = \arg\min_{X \in \mathcal{D}} d(m, m_X),
\]

where

- $\mathcal{D}$ is the set of elements of $2^\Omega$ on which we want to decide,
- $m_X$ is a categorical mass function,
- $d$ is a distance on mass functions (such as Jousselme distance)
- $m$ is the mass function coming from the function

No threshold $r$ to fit to decide on imprecise element of $2^\Omega$
The theory of belief functions provides an appropriate framework for information fusion.

Take care of conflict

- Model the conflict correctly
- Suppress the conflict
- Manage the conflict in the combination rule
- Keep the conflict until the decision step
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