Chapter 5

A class of fusion rules based on the belief redistribution to subsets or complements

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Abstract:
In this chapter we present a class of fusion rules based on the redistribution of the conflicting or even non-conflicting masses to the subsets or to the complements of the elements involved in the conflict proportionally with respect to their masses or/and cardinals. At the end, these rules are presented in a more general theoretical way including explicitly the reliability of each source of evidence. Some examples are also provided.

5.1 Introduction

In DSmT, we take very care of the model associated with the set Θ of hypotheses where the solution of the problem is assumed to belong to. In particular, the three main sets: $2^\Theta \triangleq (\Theta, \cup)$ (power set), $D^\Theta \triangleq (\Theta, \cup, \cap)$ (hyper-power set) and $S^\Theta \triangleq (\Theta, \cup, \cap, c(.))$ (super-power set) can be used depending on their ability to fit adequately with the nature of the hypotheses of the frame under consideration. These sets had been presented with examples in Chapter 1 of this volume and will be not reintroduced here. We just recall that the notion of super-power set has been introduced by Smarandache in the Chapter 8 of [13] and corresponds actually to the theoretical construction of the power set of the minimal refined frame Θ^{ref} of Θ. Actually, Θ generates S^Θ under operators $\cup$, $\cap$ and complementation $c(.)$. S^Θ is a Boolean algebra with respect to the union, intersection and complementation. Therefore working with the super-power set is equivalent to work with the power set of a minimal theoretical refined frame Θ^{ref} when the refinement is possible satisfying Shafer's model as explained in Chapter 1. Of course, when Θ already satisfies Shafer's model, the hyper-power set D^Θ and the super-power set S^Θ coincide with the classical power set 2^Θ of Θ. In general, $2^\Theta \subseteq D^\Theta \subseteq S^\Theta$. In this chapter, we introduce a new family of fusion rules based on redistribution of the conflicting (or even non-conflicting masses) to subsets or complements (RSC) for working either on the super-power set S^Θ or directly on 2^Θ whenever Shafer's model holds for Θ. This RSC family of fusion rules which uses the complementation operator $c(.)$ cannot be performed on hyper-power set D^Θ since by construction the complementation is not allowed in D^Θ.

Note that these last years, the DSmT has relaunched the studies on the combination rules especially in order to manage the conflict [1, 2, 7, 11, 12]. In [9], we proposes in the context of the DSmT some rules where not only the conflict is transferred. In [15, 16] some new combination rules are proposed to redistribute the belief to subsets or complements. Some of these rules are built similarly to the proportional conflict redistribution rules [3]. Here we extend the idea of rules based on the belief redistribution to subsets or complements.

5.2 Fusion rules based on RSC

Let $\Theta = \{\theta_1, \theta_2, \cdots, \theta_n\}$, for $n \geq 2$, be the frame of discernment of the problem under consideration, and $S^\Theta = (\Theta, \cup, \cap, c(.))$ its super-power set (see Chapter 1 for details) where $c(.)$ means the complementation operator in S^Θ. Let's denote I_t the total ignorance, i.e. $I_t \triangleq \theta_1 \cup \theta_2 \cup \cdots \cup \theta_n$. Let $m_1(.)$ and $m_2(.)$ be two normalized basic belief assignments (bba's) defined from S^Θ to $[0, 1]$. We use the conjunctive rule to first combine $m_1(.)$ with $m_2(.)$ to get $m_{12}(.)$ and then we redistribute the mass of conflict $m_{12}(X \cap Y) \neq 0$, when $X \cap Y = \emptyset$ or even when $X \cap Y$ different from the empty set, in eight ways where all denominators in these fusion rule formulas are supposed different from zero. In the sequel, we denote these rules with the acronym RSC (standing for Redistribution to Subsets or Complements) for notation convenience.

5.2.1 RSC rule no 1

If $X \cap Y = \emptyset$, then $m_{12}(X \cap Y)$ is redistributed to $c(X \cup Y)$ in the case we are not confident in X nor in Y, but we use a pessimistic redistribution. Mathematically, this RSC1 fusion rule is given by $m_{RSC1}(\emptyset) = 0$, and for all $A \in S^{\Theta} \setminus \{\emptyset, I_t\}$ by:

$$m_{RSC1}(A) = m_{12}(A) + \sum_{\substack{X, Y \in S^{\Theta} \\ X \cap Y = \emptyset \text{ and } c(X \cup Y) = A}} m_1(X)m_2(Y) \tag{5.1}$$

where $m_{12}(A) = \sum_{\substack{X, Y \in S^{\Theta} \\ X \cap Y = A}} m_1(X)m_2(Y)$ is the mass of the conjunctive consensus on A.

For the total ignorance, one has:

$$m_{RSC1}(I_t) = m_{12}(I_t) + \sum_{\substack{X, Y \in S^{\Theta} \\ X \cap Y = \emptyset \text{ and } c(X \cup Y) = \emptyset}} m_1(X)m_2(Y) \tag{5.2}$$

The second term of (5.2) takes care for the case where the complement of $X \cup Y$ is $\emptyset$ while $X \cap Y = \emptyset$. In that specific case, the mass of $X \cap Y$ is transferred to the total ignorance.

5.2.2 RSC rule no 2

If $X \cap Y = \emptyset$, then $m_{12}(X \cap Y)$ is redistributed to all subsets of $c(X \cup Y)$ proportionally with respect to their corresponding *masses* in the case we are not confident in X nor in Y, but we use an optimistic redistribution. Mathematically, this RSC2 fusion rule is given by $m_{RSC2}(\emptyset) = 0$, and for all $A \in S^{\Theta} \setminus \{\emptyset, I_t\}$ by:

$$m_{RSC2}(A) = m_{12}(A) + \sum_{\substack{X, Y \in S^{\Theta} \\ X \cap Y = \emptyset, A \in c(X \cup Y)}} \frac{m_1(X)m_2(Y)}{\sum_{Z \in c(X \cup Y) \subset S^{\Theta}} m_{12}(Z)} \cdot m_{12}(A) \tag{5.3}$$

where the denominator of the fraction is different from zero. If the denominator is zero, that fraction is discarded.

For the total ignorance, one has:

$$
\begin{aligned}
m_{RSC2}(I_t) = m_{12}(I_t) + & \sum_{\substack{X,Y \in S^\Theta \\ X\cap Y=\emptyset \text{ and } c(X\cup Y)=\emptyset}} m_1(X)m_2(Y) \\
+ & \sum_{\substack{X,Y \in S^\Theta \\ X\cap Y=\emptyset \text{ and } \sum_{Z\in c(X\cup Y)\subset S^\Theta} m_{12}(Z)=0}} m_1(X)m_2(Y)
\end{aligned}
\tag{5.4}
$$

$m_{RSC2}(I_t)$ works similarly as $m_{RSC1}(I_t)$ in the first 2 parts; in addition of this, it also assigns to I_t the masses of empty intersections whose all subsets have the mass equals to zero, so no such proportionalization is possible in $m_{RSC2}(A)$ previous formula.

5.2.3 RSC rule no 3

If $X\cap Y=\emptyset$, then $m_{12}(X\cap Y)$ is redistributed to all subsets of $c(X\cup Y)$ proportionally with respect to their corresponding *cardinals* (not masses as we did in RSC2) in the case we are not confident in X nor in Y; this is a prudent redistribution with respect to the cardinals. Mathematically, this RCS3 fusion rule is given by $m_{RSC3}(\emptyset)=0$, and for all $A\in S^\Theta\setminus\{\emptyset, I_t\}$ by:

$$
m_{RSC3}(A) = m_{12}(A) + \sum_{\substack{X,Y \in S^\Theta \\ X\cap Y=\emptyset,\, A\in c(X\cup Y)}} \frac{m_1(X)m_2(Y)}{\sum_{Z\in c(X\cup Y)\subset S^\Theta} Card(Z)} \cdot Card(A)
\tag{5.5}
$$

where the denominator of the fraction is different from zero. If the denominator is zero, that fraction is discarded.

For the total ignorance, one has:

$$
\begin{aligned}
m_{RSC3}(I_t) = m_{12}(I_t) + & \sum_{\substack{X,Y \in S^\Theta \\ X\cap Y=\emptyset \text{ and } c(X\cup Y)=\emptyset}} m_1(X)m_2(Y) \\
+ & \sum_{\substack{X,Y \in S^\Theta \\ X\cap Y=\emptyset \text{ and } \sum_{Z\in c(X\cup Y)\subset S^\Theta} Card(Z)=0}} m_1(X)m_2(Y)
\end{aligned}
\tag{5.6}
$$

5.2.4 RSC rule no 4

If $X\cap Y=\emptyset$, then $m_{12}(X\cap Y)$ is redistributed to all subsets of $c(X\cup Y)$ proportionally with respect to their corresponding *masses and cardinals* (i.e. RSC2 and RSC3 combined) in the case we are not confident in X nor in Y; this is a mixture of optimistic and prudent redistribution and this ressembles somehow to DSmP (see Chapter 3 and we could also introduce an ϵ tuning parameter). Mathematically, this RCS4 fusion rule is given by $m_{RSC4}(\emptyset) = 0$, and for all $A \in S^\Theta \setminus \{\emptyset, I_t\}$ by:

$$m_{RSC4}(A) = m_{12}(A) + \sum_{\substack{X, Y \in S^\Theta \\ X \cap Y = \emptyset,\, A \in c(X \cup Y)}} \frac{m_1(X)m_2(Y)}{\sum_{Z \in c(X \cup Y) \subset S^\Theta} [m_{12}(Z) + Card(Z)]} \cdot [m_{12}(A) + Card(A)] \tag{5.7}$$

where the denominator of the fraction is different from zero. If the denominator is zero, that fraction is discarded.

For the total ignorance, one has:

$$m_{RSC4}(I_t) = m_{12}(I_t) + \sum_{\substack{X, Y \in S^\Theta \\ X \cap Y = \emptyset \text{ and } c(X \cup Y) = \emptyset}} m_1(X)m_2(Y) + \sum_{\substack{X, Y \in S^\Theta \\ X \cap Y = \emptyset \text{ and } \sum_{Z \in c(X \cup Y) \subset S^\Theta} [m_{12}(Z) + Card(Z)] = 0}} m_1(X)m_2(Y) \tag{5.8}$$

5.2.5 RSC rule no 5

If $X \cap Y = \emptyset$, then $m_{12}(X \cap Y)$ is redistributed to X and Y proportionally with respect to their corresponding *cardinals*. Mathematically, this RSC5 fusion rule is given by $m_{RSC5}(\emptyset) = 0$, and for all $A \in S^\Theta \setminus \{\emptyset\}$ by:

$$m_{RSC5}(A) = m_{12}(A) + \sum_{\substack{X \in S^\Theta \\ X \cap A = \emptyset}} \frac{m_1(X)m_2(A) + m_1(A)m_2(X)}{Card(X) + Card(A)} \cdot Card(A) \tag{5.9}$$

where the denominator of the fraction is different from zero. If the denominator is zero, that fraction is discarded.

5.2.6 RSC rule no 6

If $X \cap Y = \emptyset$, then $m_{12}(X \cap Y)$ is redistributed to all subsets of $X \cup Y$ proportionally with respect to their corresponding *cardinals*. Mathematically, this RSC6 fusion rule is given by $m_{RSC6}(\emptyset) = 0$, and for all $A \in S^\Theta \setminus \{\emptyset\}$ by:

$$m_{RSC6}(A) = m_{12}(A) + \sum_{\substack{X,Y \in S^\Theta \\ X \cap Y = \emptyset \\ A \subseteq X \cup Y}} \frac{m_1(X)m_2(Y)}{\sum\limits_{Z \in S^\Theta, Z \subseteq X \cup Y} Card(Z)} \cdot Card(A) \tag{5.10}$$

where the denominator of the fraction is different from zero. If the denominator is zero, that fraction is discarded.

5.2.7 RSC rule no 7

If $X \cap Y = \emptyset$, then $m_{12}(X \cap Y)$ is redistributed to X and Y proportionally with respect to their corresponding *cardinals and masses*. Mathematically, this RSC7 fusion rule is given by $m_{RSC7}(\emptyset) = 0$, and for all $A \in S^\Theta \setminus \{\emptyset\}$ by:

$$m_{RSC7}(A) = m_{12}(A) + \sum_{\substack{X \in S^\Theta \\ X \cap A = \emptyset}} \frac{m_1(X)m_2(A) + m_1(A)m_2(X)}{Card(X) + Card(A) + m_{12}(X) + m_{12}(A)} \cdot [Card(A) + m_{12}(A)] \tag{5.11}$$

where the denominator of the fraction is different from zero. If the denominator is zero, that fraction is discarded.

5.2.8 RSC rule no 8

If $X \cap Y = \emptyset$, then $m_{12}(X \cap Y)$ is redistributed to all subsets of $X \cup Y$ proportionally with respect to their corresponding *cardinals and masses*. Mathematically, this new fusion rule (denoted RSC8) is given by $m_{RSC8}(\emptyset) = 0$, and for all $A \in S^\Theta \setminus \{\emptyset\}$ by:

$$m_{RSC8}(A) = m_{12}(A) + \sum_{\substack{X,Y \in S^\Theta \\ X \cap Y = \emptyset \\ A \subseteq X \cup Y}} \frac{m_1(X)m_2(Y)}{\sum\limits_{Z \in S^\Theta, Z \subseteq X \cup Y} Card(Z) + m_{12}(Z)} \cdot [Card(A) + m_{12}(A)] \tag{5.12}$$

where the denominator of the fraction is different from zero. If the denominator is zero, that fraction is discarded.

5.2.9 Remarks

We can generalize all these previous formulas for any $s \geq 2$, where s is the number of sources. We can adjust all these formulas for the case when $X \cap Y \neq \emptyset$ but we still want to transfer $m_{12}(X \cap Y)$ to subsets of $c(X \cup Y)$, or to subset of $X \cup Y$, or to both groups of subsets, but we need to have a justification for these.

In choosing a fusion rule, among so many, we apply the following criteria:

a) Reliability of sources of information $m_i(.)$: Are they all reliable or not ? In what percentage is reliable each source ?

b) Confidence in the hypotheses of the frame of discernment and in elements of S^Θ: Are we confident in all of them ? In what percentage are we confident in each of them ?

c) Optimistic,pessimistic, or medium redistribution of the conflicting masses - depending on user's experience.

5.3 A new class of RSC fusion rules

Using the conjunctive rule, let's denote:

$$m_\cap(A) \equiv m_{12}(A) = [m_1 \oplus m_2](A) = \sum_{\substack{X, Y \in S^\Theta \\ X \cap Y = A}} m_1(X) m_2(Y)$$

For $A \in S^\Theta \setminus \{\emptyset, I_t\}$, we have the following new class of fusion rule (denoted $CRSC_c$) for transferring the conflicting masses only:

$$\begin{aligned} m_{CRSC_c}(A) = m_\cap(A) + [\alpha \cdot m_\cap(A) + \beta \cdot Card(A) + \gamma \cdot f(A)] \cdot \\ \cdot \sum_{\substack{X, Y \in S^\Theta \\ X \cap Y = \emptyset \\ A \subseteq M}} \frac{m_1(X) m_2(Y)}{\sum_{Z \in S^\Theta, Z \subseteq M} [\alpha \cdot m_\cap(Z) + \beta \cdot Card(Z) + \gamma \cdot f(Z)]} \end{aligned} \tag{5.13}$$

where M can be $c(X \cup Y)$, or a subset of $c(X \cup Y)$, or $X \cup Y$, or a subset of $X \cup Y$; $\alpha, \beta, \gamma \in \{0, 1\}$ but $\alpha + \beta + \gamma \neq 0$; in a weighted way we can take $\alpha, \beta, \gamma \in [0, 1]$ also with $\alpha + \beta + \gamma \neq 0$; $f(X)$ is a function of X, i.e. another parameter that the mass of X is directly proportionally with respect to; $card(X)$ is the cardinal of X.

And $m_{CRSC_c}(I_t)$ is given by:

$$m_{CRSC_c}(I_t) = m_\cap(I_t) + \sum_{\substack{X, Y \in S^\Theta \\ \{X \cap Y = \emptyset \text{ and } M = \emptyset\} \\ \text{or} \{X \cap Y = \emptyset \text{ and } Den(Z) = 0\}}} m_1(X) m_2(Y) \tag{5.14}$$

where $Den(Z) \triangleq \sum_{Z \in S^\Theta, Z \subseteq M}[\alpha \cdot m_\cap(Z) + \beta \cdot Card(Z) + \gamma \cdot f(Z)]$.

In $m_{CRSC_c}(.)$ formula if we replace: $\alpha = 0$ or 1, $\beta = 0$ or 1, $\gamma = 0$, $M = c(X \cup Y)$ or $X \cup Y$, or $\{\{X\}, \{Y\}\}$, we obtain nine fusion rules including the previous 2-8 rules. For $\alpha = 1$, $\beta = 0$, $\gamma = 0$, $M = \{c(X), X \cap Y, c(Y)\}$, we obtain one of Yamada's rules [15, 16] and discussed in [3]. For $\alpha = 1$, $\beta = 0$, $\gamma = 0$, $M = \{X \cap Y, X \cup Y\}$, we obtain another one of Yamada's rules.

5.4 A general formulation

Let $\Theta = \{\theta_1, \theta_2, \cdots, \theta_n\}$, for $n \geq 2$ be the frame of discernment, and $S^\Theta = (\Theta, \cup, \cap, c(.))$ its super-power set. The element Θ (also denoted I_t) represents the total ignorance. When the elements θ_i are exclusive two by two S^Θ reduces to the classical power set 2^Θ, otherwise $S^\Theta \equiv 2^{\Theta^{ref}}$ if Θ is refinable and $|S^\Theta| = 2^{2^{|\Theta|}-1}$ (see chapter 1 for details and examples). $c(X)$ means the complement of X in S^Θ. $S^\Theta = (\Theta, \cup, \cap, c(.))$ can also be written as:

$$S^\Theta = D^{\Theta \cup \Theta_c} = 2^{\Theta^{ref}} \tag{5.15}$$

where Θ_c represents the set of complements of the the elements of Θ in 2^Θ.

Example: Let's consider the example given in section 1.2.1 of Chapter 1 using $\Theta = \{\theta_1, \theta_2\}$ with $\theta_1 \cap \theta_2 \neq \emptyset$. According to the definition and the construction of the super-power set, one obtains directly

$$S^\Theta = (\Theta, \cup, \cap, c(.)) = \{\emptyset, \theta_1 \cap \theta_2, \theta_1, \theta_2, \theta_1 \cup \theta_2, c(\theta_1 \cap \theta_2), c(\theta_1), c(\theta_2)\}$$

If we consider both sets $\Theta = \{\theta_1, \theta_2\}$ and $\Theta_c = \{c(\theta_1), c(\theta_2)\}$, then $\Theta \cup \Theta_c = \{\theta_1, \theta_2, c(\theta_1), c(\theta_2)\}$ with the integrity constraints $\theta_1 \cap c(\theta_1) = \emptyset$, $\theta_2 \cap c(\theta_2) = \emptyset$, $c(\theta_1) \cap c(\theta_2) = \emptyset$ and $(\theta_1 \cap \theta_2) \cap (c(\theta_1) \cap c(\theta_2)) = \emptyset$. The hyper-power set $D^{\Theta \cup \Theta_c}$ taking into account all integrity constraints is then given by:

$$D^{\Theta \cup \Theta_c} = \{\emptyset, \theta_1 \cap \theta_2, \theta_1, \theta_2, \theta_1 \cup \theta_2, c(\theta_1) \cup c(\theta_2), c(\theta_1), c(\theta_2)\}$$

but since $c(\theta_1) \cup c(\theta_2) = c(\theta_1 \cap \theta_2)$ (Morgan's law), one sees that

$$D^{\Theta \cup \Theta_c} = (\{\Theta \cup \Theta_c\}, \cup, \cap) = \{\emptyset, \theta_1 \cap \theta_2, \theta_1, \theta_2, \theta_1 \cup \theta_2, c(\theta_1 \cap \theta_2), c(\theta_1), c(\theta_2)\} = S^\Theta$$

Moreover, if we consider the following theoretical refined frame Θ^{ref} built from Θ as follows: $\Theta^{ref} = \{c(\theta_1), \theta_1 \cap \theta_2, c(\theta_2)\}$ where now all elements of Θ^{ref} are truly exclusive, then $2^{\Theta^{ref}} = \{\emptyset, c(\theta_1), \theta_1 \cap \theta_2, c(\theta_2), c(\theta_1) \cup (\theta_1 \cap \theta_2), c(\theta_1) \cup c(\theta_2), (\theta_1 \cap \theta_2) \cup c(\theta_2), c(\theta_1) \cup (\theta_1 \cap \theta_2) \cup c(\theta_2)\}$ which can be simplified since by construction of the refined frame, $\theta_1 = (\theta_2) \cup (\theta_1 \cap \theta_2)$, $\theta_2 = (\theta_1) \cup (\theta_1 \cap \theta_2)$ and $\theta_1 \cup \theta_2 = (\theta_1) \cup (\theta_1 \cap \theta_2) \cup (\theta_2)$:

$$2^{\Theta^{ref}} = \{\emptyset, c(\theta_1), \theta_1 \cap \theta_2, c(\theta_2), \theta_2, c(\theta_1) \cup c(\theta_2), \theta_1, \theta_1 \cup \theta_2\}$$

After rearranging the list of elements of $2^{\Theta^{ref}}$ and since $c(\theta_1) \cup c(\theta_2) = c(\theta_1 \cap \theta_2)$, one finally sees that

$$2^{\Theta^{ref}} = (\Theta^{ref}, \cup) = \{\emptyset, \theta_1 \cap \theta_2, \theta_1, \theta_2, \theta_1 \cup \theta_2, c(\theta_1 \cap \theta_2), c(\theta_1), c(\theta_2)\} = S^{\Theta}$$

The proposed class of fusion rules is based on a proportional conflict transfer. When there is no conflict between experts the conjunctive rule is used, otherwise the masses of conflicts resulting from conjunctive fusion of experts for incompatible propositions of (super) power set is redistributed on some compatible propositions through different mechanisms which give rise to different fusion rules as explained in the sequel. The use of the conjunctive rule assumes that the experts are reliable, or the reliability of each expert is known and taken into account in the mass values. We denote the set of intersections/conjunctions by: $S_{\cap} = \{X \in S^{\Theta} | X = Y \cap Z, \text{ where } Y, Z \in S^{\Theta} \setminus \{\emptyset\}\}$ where all propositions are expressed in their canonical form and where Xcontains at least an $\cap$ symbol in its expression. For example, $A \cap A \notin S_{\cap}$ since $A \cap A$ is not in a canonical form and $A \cap A = A$. Also $(A \cap B) \cap B$ is not a canonical form but $(A \cap B) \cap B = A \cap B \in S_{\cap}$.

Let $S_{\cap}^{\emptyset}$ be the set of all empty intersections from $S_{\cap}$ (i.e. the set of exclusivity constraints), and $S_{\cap}^{non\emptyset}$ the set of all non-empty intersections from $S_{\cap}$, and $S_{\cap,r}^{non\emptyset}$ the set of all non-empty intersections from $S_{\cap}^{non\emptyset}$ whose masses are redistributed to other sets/propositions. The set $S_{\cap,r}^{non\emptyset}$ highly depends on the model for the frame of the application under consideration.

5.4.1 A general formula for the class of RSC fusion rules

For $A \in (S^{\Theta} \setminus S_{\cap}^{non\emptyset}) \setminus \{\emptyset, \Theta\}$, we propose the general formula for the redistribution of conflict and non-conflict to subsets or complements class of rules for the fusion of masses of belief for two sources of evidence:

$$m_{CRSC}(A) = m_{\cap}(A) + \sum_{\substack{X, Y \in S^{\Theta} \\ \{X \cap Y = \emptyset, A \in T(X,Y)\} \\ \text{or } \{X \cap Y \in S_{\cap,r}^{non\emptyset}, A \in T'(X,Y)\}}} f(A) \frac{m_1(X) m_2(Y)}{\sum_{Z \in T(X,Y)} f(Z)} \tag{5.16}$$

and for $A = \Theta$:

$$m_{CRSC}(\Theta) = m_{\cap}(\Theta) + \sum_{\substack{X, Y \in S^{\Theta} \\ X \cap Y = \emptyset, \\ \{T(X,Y) = \emptyset \text{ or } \sum_{Z \in T(X,Y)} f(Z) = 0\}}} m_1(X) m_2(Y) \tag{5.17}$$

where f is a mapping from S^{Θ} to $\mathbb{R}^+$. For example, we can choose $f(X) = m_{\cap}(X)$, $f(X) = |X|$, $f^T(X) = \frac{|X|}{|T(X,Y)|}$, or $f(x) = m_{\cap}(X) + |X|$, etc. The function T specifies a subset of S^{Θ}, for example $T(X,Y) = \{c(X \cup Y)\}$, or $T(X,Y) = \{X \cup Y\}$ or can specify a set of subsets of S^{Θ}. For example, $T(X,Y) = \{A \subset c(X \cup Y)\}$, or $T(X,Y) = \{A \subset X \cup Y\}$. The function T' is a subset of S^Θ, for example $T'(X,Y) = \{X \cup Y\}$, or T' is a subset of $X \cup Y$, etc.

It is important to highlight that in formulas (5.13)-(5.14) one transfers only the conflicting masses, whereas the formulas (5.16)-(5.17) are more general since one transfers the conflicting masses or the non-conflicting masses as well depending on the preferences of the fusion system designer. The previous formulas can be directly extended for any $s \geq 2$ sources of evidence as follows: For $A \in (S^\Theta \setminus S_\cap^{non\emptyset}) \setminus \{\emptyset, \Theta\}$ we have:

$$m_{CRSC}(A) = m_\cap(A) + \sum_{\substack{X_1, \cdots, X_s \in S^\Theta \\ \{\cap_{i=1}^s X_i = \emptyset, A \in T(X_1, \cdots, X_s)\} \\ \text{or } \{\cap_{i=1}^s X_i \in S_{\cap,r}^{non\emptyset}, A \in T'(X_1, \cdots, X_s)\}}} f(A) \frac{\prod_{i=1}^s m_i(X_i)}{\sum_{Z \in T(X_1, \cdots, X_s)} f(Z)} \tag{5.18}$$

and for $A = \Theta$:

$$m_{CRSC}(\Theta) = m_\cap(\Theta) + \sum_{\substack{X_1, \cdots, X_s \in S^\Theta \\ \cap_{i=1}^s X_i = \emptyset, \\ \{T(X_1, \cdots, X_s) = \emptyset \text{ or } \sum_{Z \in T(X_1, \cdots, X_s)} f(Z) = 0\}}} \prod_{i=1}^s m_i(X_i) \tag{5.19}$$

This class of rules of combination is a particular case of the rule given in [6].

5.4.2 Example

We illustrate here the previous general formulas on a simple example corresponding to the hybrid model given in the figure 5.1. $\Theta = \{A, B, C, D\}$, with $A \cap B \neq \emptyset$ and all other intersections are empty.

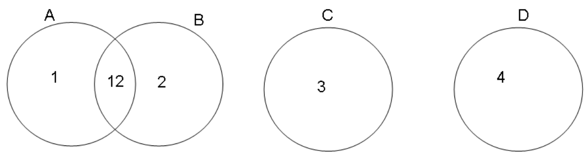

Figure 5.1: Hybrid model.

Let's consider two sources of evidence with their masses of belief $m_1(.)$ and $m_2(.)$ given in the following table:

	m_1	m_2	$m_\cap$
A	0.2	0.4	0.18
B	0.3	0.2	0.13
C	0.1	0.2	0.07
D	0.2	0.1	0.06
$A \cup B \cup C \cup D$	0.2	0.1	0.02
$A \cap B \neq \emptyset$			0.16
$A \cap C = \emptyset$			0.08
$A \cap D = \emptyset$			0.10
$B \cap C = \emptyset$			0.08
$B \cap D = \emptyset$			0.07
$C \cap D = \emptyset$			0.05

Let's apply the general formula (5.16)-(5.17) with different choices of the function $f(.)$ and $T(X,Y)$:

- **RSC2 rule**: we take $f(A) = m_\cap(A)$, $T(X,Y) = 2^{c(X\cup Y)}$, $T(X,Y) = \emptyset$. $m_\cap(A \cap C) = 0.08$ is transfered to all subsets of $c(A \cup C)$ proportionally with respect to their masses, but D is a subset whose mass is not zero; so the whole conflicting mass 0.08 is transfered to D. Similarly:

  - $m_\cap(A \cap D) = 0.10$ is transfered to C only,
  - $m_\cap(B \cap C) = 0.08$ is transfered to D only,
  - $m_\cap(B \cap D) = 0.07$ is transfered to C only,

  But $m_\cap(C \cap D) = 0.05$ is transfered to A and B which are subsets of non-zero mass of $c(C \cup D)$ proportionally with respect to their corresponding masses 0.18 and 0.13 respectively. We obtain:

	A	B	C	D	$A \cup B \cup C \cup D$	$A \cap B \neq \emptyset$
m_{RSC2}	0.21	0.15	0.24	0.22	0.02	0.16

- **RSC3 rule**: we take $f(A) = |A|$, $T(X,Y) = 2^{c(X\cup Y)}$, $T(X,Y) = \emptyset$.
  $m_\cap(A \cap C) = 0.08$ is transfered to the parts 2, 4 and $2 \cup 4$ proportionally with respect to their cardinals: 1, 1, 2 respectively. Hence the parts 2 and 4 receive 0.02 and $2 \cup 4$ 0.04. Similarly

  - $m_\cap(A \cap D) = 0.10$ is transfered to 2, 3 and $2 \cup 3$ with respectively 0.025, 0.025 and 0.05.
  - $m_\cap(B \cap C) = 0.08$ is transfered to 1, 4 and $1 \cup 4$ with respectively 0.02, 0.02 and 0.04.
  - $m_\cap(B \cap D) = 0.07$ is transfered to 1, 3 and $1 \cup 3$ with respectively 0.0175, 0.0175 and 0.035.
  - $m_\cap(C \cap D) = 0.05$ is transfered to 1, 2, 12, $1\cup2$, $1\cup12$,$2\cup12$, $1\cup2\cup12$ with respectively 0.004166, 0.004166, 0.004166, 0.008333, 0.008333, 0.008333, 0.012503.

- **RSC4 rule**: we take $f(A) = m_{\cap}(A) + |A|$, $T(X,Y) = 2^{c(X \cup Y)}$ and $T(X,Y) = \emptyset$.
- **RSC5 rule**: we take $f(A) = |A|$, $T(X,Y) = \{X,Y\}$ and $T(X,Y) = \emptyset$.
- **RSC6 rule**: we take $f(A) = |A|$, $T(X,Y) = 2^{X \cup Y}$ and $T(X,Y) = \emptyset$.
- **RSC7 rule**: we take $f(A) = m_{\cap}(A) + |A|$, $T(X,Y) = \{X,Y\}$ and $T(X,Y) = \emptyset$.
- **RSC8 rule**: we take $f(A) = m_{\cap}(A) + |A|$, $T(X,Y) = 2^{X \cup Y}$ and $T(X,Y) = \emptyset$.

5.5 A general formulation including reliability

A general fusion formulation including explicitly the reliabilities of the sources of evidence is given by the following formula: for all $A \in S^{\Theta}$, one has

$$m(X) = \sum_{\mathbf{Y} \in (S^{\Theta})^s} \prod_{j=1}^{s} m_j(Y_j) w(X, \mathbf{m}(\mathbf{Y}), T(\mathbf{Y}), \alpha), \tag{5.20}$$

where $\mathbf{Y} = (Y_1, \cdots, Y_s)$ are the responses of the s experts and $m_j(Y_j)$ their associated mass of belief; α is a matrix of terms α_{ij} of the reliability of the expert j for the element i of S^{Θ}, and $\mathbf{Y}, T(\mathbf{Y})$ is the set of subsets of S^{Θ} on which we can transfer the masses $m_j(Y_j)$ for the given $\mathbf{Y}$ vector. In this general formulation, the argument $\mathbf{Y}$ of the transfer function $T(.)$ is a vector of dimension s, whereas we did use the notation $T(X,Y)$ in the two sources case in eq. (5.17).

5.5.1 Examples

We show how to retrieve the principal rules of combinations from the previous general formula (5.20):

- **Conjunctive rule**: It is obtained from (5.20) by taking

$$w(X, \mathbf{m}(\mathbf{Y}), T(\mathbf{Y}), \alpha) = 1 \quad \text{if} \quad \cap_{j=1}^{s} Y_j = X \tag{5.21}$$

  $T(\mathbf{Y}) = \cap_{j=1}^{s} Y_j$ and we do not consider α.

- **Disjunctive rule** in [4] : It is obtained from (5.20) by taking

$$w(X, \mathbf{m}(\mathbf{Y}), T(\mathbf{Y}), \alpha) = 1 \quad \text{if} \quad \cup_{j=1}^{s} Y_j = X \tag{5.22}$$

  $T(\mathbf{Y}) = \cup_{j=1}^{s} Y_j$ and we do not consider α.

- **Dubois & Prade rule** in [5]: It is obtained from (5.20) by taking

$$w(X, \mathbf{m}(\mathbf{Y}), T(\mathbf{Y}), \alpha) = \begin{cases} 1 & \text{if } \cap_{j=1}^{s} Y_j = X \text{ and } X \neq \emptyset \\ 1 & \text{if } \cup_{j=1}^{s} Y_j = X \text{ and } X = \emptyset \end{cases} \tag{5.23}$$

  $T(\mathbf{Y}) = \{\cap_{j=1}^{s} Y_j, \cup_{j=1}^{s} Y_j\} \setminus \emptyset$ and we do not consider α.

- **PCR5 rule** introduced in [13], from the equation given in [7]: It is obtained from (5.20) by taking

  a) $w(X, \mathbf{m}(\mathbf{Y}), T(\mathbf{Y}), \alpha) = 1$ whenever $\cap_{j=1}^{s} Y_j = X$, and $X \neq \emptyset$.

  b) and whenever $Y_i = X, i = 1, \cdots, s$, and $\cap_{j=1}^{s} Y_j = \emptyset$,

$$w(X, \mathbf{m}(\mathbf{Y}), T(\mathbf{Y}), \alpha) = \frac{\displaystyle\sum_{i=1}^{M} m_i(X)}{\displaystyle\prod_{j=1}^{s} m_j(Y_j)}$$

$$\times \frac{\left(\displaystyle\prod_{j=1}^{M-1} m_{\sigma_i(j)}(Y_{\sigma_i(j)}) \mathbb{1}_{j>i}\right) \displaystyle\prod_{Y_{\sigma_i(j)}=X} m_{\sigma_i(j)}(Y_{\sigma_i(j)})}{\displaystyle\sum_{Z \in \{X, Y_{\sigma_i(1)}, \ldots, Y_{\sigma_i(M-1)}\}} \displaystyle\prod_{Y_{\sigma_i(j)}=Z} \left(m_{\sigma_i(j)}(Y_{\sigma_i(j)}) \cdot \xi(X{=}Z, m_i(X))\right)} \tag{5.24}$$

  where:

$$\begin{cases} \sigma_i(j) = j, \text{ if } j < i, \\ \sigma_i(j) = j + 1, \text{ if } j \geq i, \end{cases} \tag{5.25}$$

$$\begin{cases} \xi(B, x) = x & \text{if } B \text{ is true,} \\ \xi(B, x) = 1 & \text{if } B \text{ is false.} \end{cases} \tag{5.26}$$

  and $T(\mathbf{Y}) = \{\cap_{j=1}^{s} Y_j, Y_1, \cdots, Y_s\} \setminus \emptyset$ and we do not consider α.

- **PCR6 rule** in [7]: It is obtained from (5.20) by taking

  a) $w(X, \mathbf{m}(\mathbf{Y}), T(\mathbf{Y}), \alpha) = 1$ whenever $\cap_{j=1}^{s} Y_j = X$, and $X \neq \emptyset$,

  b) and whenever $Y_i = X, i = 1, \cdots, s$, and $\cap_{j=1}^{s} Y_j = \emptyset$,

$$w(X, \mathbf{m}(\mathbf{Y}), T(\mathbf{Y}), \alpha) = \frac{\displaystyle\sum_{i=1}^{M} m_i(X)}{m_i(X) + \displaystyle\sum_{j=1}^{M-1} m_{\sigma_i(j)}(Y_{\sigma_i(j)})} \tag{5.27}$$

  where:

$$\begin{cases} \sigma_i(j) = j, \text{ if } j < i, \\ \sigma_i(j) = j + 1, \text{ if } j \geq i, \end{cases} \tag{5.28}$$

  $T(\mathbf{Y}) = \{\cap_{j=1}^{s} Y_j, Y_1, \cdots, Y_s\} \setminus \emptyset$ and we do not consider α.

- **RSC1 rule**: It is obtained from (5.20) by taking

$$w(X, \mathbf{m}(\mathbf{Y}), T(\mathbf{Y}), \alpha) = \begin{cases} 1 \text{ if } \cap_{j=1}^{s} Y_j = X \text{ and } X \neq \emptyset \\ 1, \text{ if } c\left(\cap_{j=1}^{s} Y_j\right) = X \text{ and } \cup_{j=1}^{s} Y_j = \emptyset \end{cases} \tag{5.29}$$

$T(\mathbf{Y}) = \{\cap_{j=1}^{s} Y_j, c\left(\cup_{j=1}^{s} Y_j\right)\} \setminus \emptyset$ and we do not consider α.

- **RSC2 rule**: It is obtained from (5.20) by taking

  a) $w(X, \mathbf{m}(\mathbf{Y}), T(\mathbf{Y}), \alpha) = 1$ whenever $\cap_{j=1}^{s} Y_j = X$, and $X \neq \emptyset$.

  b)
$$w(X, \mathbf{m}(\mathbf{Y}), T(\mathbf{Y}), \alpha) = \frac{m_\cap(X)}{\sum\limits_{Z \subseteq c\left(\cup_{j=1}^{s} Y_j\right)} m_\cap(Z)} \tag{5.30}$$

  whenever $X \in 2^{c\left(\cup_{j=1}^{s} Y_j\right)}$, $\cap_{j=1}^{s} Y_j = \emptyset$, $c\left(\cup_{j=1}^{s} Y_j\right) \neq \emptyset$ and

$$\sum_{Z \subseteq c\left(\cup_{j=1}^{s} Y_j\right)} m_\cap(Z) \neq 0.$$

  c) $w(X, \mathbf{m}(\mathbf{Y}), T(\mathbf{Y}), \alpha) = 1$ whenever $X = \Theta$, $\cap_{j=1}^{s} Y_j = \emptyset$, and

$$\{c\left(\cup_{j=1}^{s} Y_j\right) = \emptyset \quad \text{or} \quad \sum_{Z \subseteq c\left(\cup_{j=1}^{s} Y_j\right)} m_\cap(Z) = 0\}.$$

$T(\mathbf{Y}) = \{\cap_{j=1}^{s} Y_j, \{2^{c\left(\cup_{j=1}^{s} Y_j\right)}\}, \Theta\} \setminus \emptyset$ and we do not consider α.

- **RSC3 rule**: It is obtained from (5.20) by taking

  a) $w(X, \mathbf{m}(\mathbf{Y}), T(\mathbf{Y}), \alpha) = 1$ whenever $\cap_{j=1}^{s} Y_j = X$ and $X \neq \emptyset$.

  b)
$$w(X, \mathbf{m}(\mathbf{Y}), T(\mathbf{Y}), \alpha) = \frac{|X|}{\sum\limits_{Z\left(\cup_{j=1}^{s} Y_j\right)} |Z|} \tag{5.31}$$

  if $X \in 2^{c\left(\cup_{j=1}^{s} Y_j\right)}$, $m_\cap(X) \neq 0$, $c\left(\cup_{j=1}^{s} Y_j\right) \neq \emptyset$ and $\cap_{j=1}^{s} Y_j = \emptyset$.

  c) $w(X, \mathbf{m}(\mathbf{Y}), T(\mathbf{Y}), \alpha) = 1$ if $X = \Theta$, $c\left(\cup_{j=1}^{s} Y_j\right) = \emptyset$ and $\cap_{j=1}^{s} Y_j = \emptyset$.

$T(\mathbf{Y}) = \{\cap_{j=1}^{s} Y_j, \{2^{c\left(\cup_{j=1}^{s} Y_j\right)}\}, \Theta\} \setminus \emptyset$ and we do not consider α.
Remark: $\sum_{Z\left(\cup_{j=1}^{s} Y_j\right)} |Z| \neq 0$ because $c\left(\cup_{j=1}^{s} Y_j\right) \neq \emptyset$.

- **RSC4 rule**: It is obtained from (5.20) by taking

  a) $w(X, \mathbf{m}(\mathbf{Y}), T(\mathbf{Y}), \alpha) = 1$ whenever $\cap_{j=1}^{s} Y_j = X$ and $X \neq \emptyset$.

b)

$$w(X, \mathbf{m}(\mathbf{Y}), T(\mathbf{Y}), \alpha) = \frac{m_\cap(X) + |X|}{\displaystyle\sum_{Z \subseteq c\left(\cup_{j=1}^{s} Y_j\right)} m_\cap(Z) + |Z|} \tag{5.32}$$

if $X \in 2^{c\left(\cup_{j=1}^{s} Y_j\right)}$, $\cap_{j=1}^{s} Y_j = \emptyset$ and $c\left(\cup_{j=1}^{s} Y_j\right) \neq \emptyset$.

c) $w(X, \mathbf{m}(\mathbf{Y}), T(\mathbf{Y}), \alpha) = 1$ if $X = \Theta$, $\cap_{j=1}^{s} Y_j = \emptyset$ and $c\left(\cup_{j=1}^{s} Y_j\right) = \emptyset$.

$T(\mathbf{Y}) = \{\cap_{j=1}^{s} Y_j, \{2^{c\left(\cup_{j=1}^{s} Y_j\right)}\}, \Theta\} \setminus \emptyset$ and we do not consider α.

- **RSC5 rule**: It is obtained from (5.20) by taking

a) $w(X, \mathbf{m}(\mathbf{Y}), T(\mathbf{Y}), \alpha) = 1$ whenever $\cap_{j=1}^{s} Y_j = X$ and $X \neq \emptyset$.

b)

$$w(X, \mathbf{m}(\mathbf{Y}), T(\mathbf{Y}), \alpha) = \frac{|X|}{\displaystyle\sum_{j=1}^{s} |Y_j|} \tag{5.33}$$

if $Y_i = X, i = 1, \cdots, s$, and $\cap_{j=1}^{s} Y_j = \emptyset$.

$T(\mathbf{Y}) = \{\cap_{j=1}^{s} Y_j, Y_1, \cdots, Y_s\} \setminus \emptyset$ and we do not consider α.

- **RSC6 rule**: It is obtained from (5.20) by taking

a) $w(X, \mathbf{m}(\mathbf{Y}), T(\mathbf{Y}), \alpha) = 1$ whenever $\cap_{j=1}^{s} Y_j = X$ and $X \neq \emptyset$.

b)

$$w(X, \mathbf{m}(\mathbf{Y}), T(\mathbf{Y}), \alpha) = \frac{|X|}{\displaystyle\sum_{Z \subseteq \cup_{j=1}^{s} Y_j} |Z|} \tag{5.34}$$

if $X \in 2^{\cup_{j=1}^{s} Y_j}$, and $\cap_{j=1}^{s} Y_j = \emptyset$.

$T(\mathbf{Y}) = \{\cap_{j=1}^{s} Y_j, \{2^{\cup_{j=1}^{s} Y_j}\}\} \setminus \emptyset$ and we do not consider α.

- **RSC7 rule**: It is obtained from (5.20) by taking

a) $w(X, \mathbf{m}(\mathbf{Y}), T(\mathbf{Y}), \alpha) = 1$ whenever $\cap_{j=1}^{s} Y_j = X$ and $X \neq \emptyset$.

b)

$$w(X, \mathbf{m}(\mathbf{Y}), T(\mathbf{Y}), \alpha) = \frac{m_\cap(X) + |X|}{\displaystyle\sum_{j=1}^{s} m_\cap(Y_j) + |Y_j|} \tag{5.35}$$

if $Y_i = X, i = 1, \cdots, s$, and $\cap_{j=1}^{s} Y_j = \emptyset$.

$T(\mathbf{Y}) = \{\cap_{j=1}^{s} Y_j, Y_1, \cdots, Y_s\} \setminus \emptyset$ and we do not consider α.

- **RSC8 rule**: It is obtained from (5.20) by taking

  a) $w(X, \mathbf{m}(\mathbf{Y}), T(\mathbf{Y}), \alpha) = 1$ whenever $\cap_{j=1}^{s} Y_j = X$ and $X \neq \emptyset$.

  b)
$$w(X, \mathbf{m}(\mathbf{Y}), T(\mathbf{Y}), \alpha) = \frac{m_{\cap}(X) + |X|}{\displaystyle\sum_{Z \subseteq \cup_{j=1}^{s} Y_j} m_{\cap}(Z) + |Z|} \tag{5.36}$$

  if $X \in 2^{\cup_{j=1}^{s} Y_j}$, $\cap_{j=1}^{s} Y_j = \emptyset$, and $\cup_{j=1}^{s} Y_j \neq \emptyset$.

  c) $w(X, \mathbf{m}(\mathbf{Y}), T(\mathbf{Y}), \alpha) = 1$ if $X = \Theta$, $\cup_{j=1}^{s} Y_j = \emptyset$.

  $T(\mathbf{Y}) = \{\cap_{j=1}^{s} Y_j, \{2^{\cup_{j=1}^{s} Y_j}\}, \Theta\} \setminus \emptyset$ and we do not consider α.

5.6 A new rule including reliability

The idea we propose here consists in transferring the mass on $D^T \setminus \{\emptyset\}$, with $T = \{Y_1, \cdots, Y_s, c(Y_1), \cdots, c(Y_s)\}$, according with respect to their mass and reliability α_{ij}, $i = 1, \cdots, s$ and $j = 1, \cdots, |S^\Theta|$ an arbitrary order on S^Θ. Hence with the previous notations, $T(\mathbf{Y}) = D^T \setminus \{\emptyset\}$.

5.6.1 The fusion of two experts including their reliability

We first explain the idea for two experts given a basic belief assignment respectively on X and Y. Hence $T(X, Y) = D^{\{X, Y, c(X), c(Y)\}} \setminus \{\emptyset\}$. We note that $X \cup c(X) = Y \cup c(Y) = \Theta$ and $c(X) \cap c(Y) = c(X \cup Y)$ and if $X \cap Y = \emptyset$: $X \cap c(Y) = X$, $Y \cap c(X) = Y$ and $c(X) \cup c(Y) = \Theta$. Hence:

$$\begin{aligned} T(X, Y) = \{&X, Y, X \cap Y, X \cup Y, c(X), c(X) \cap Y, c(X) \cup Y, c(Y), \\ &c(Y) \cap X, c(Y) \cup X, c(X) \cup c(Y), c(X) \cap c(Y), \Theta\} \end{aligned}$$

If $X \cap Y \neq \emptyset$, and if the reliability $\alpha_{1X} = \alpha_{1Y} = 1$, and if $m_1(X) = m_2(Y) = 1$ then all the belief must be given on $X \cap Y$. If the reliability $\alpha_{1X} = \alpha_{1Y} = 1$ but $m_1(X) \neq 1$ and $m_2(Y) \neq 1$, then the experts are not sure and a part of the mass $m_1(X).m_2(Y)$ can also be transfered on $X \cup Y$. If for example $\alpha_{1X} = 0$ then we should also transfer mass on $c(X)$. If $X \cap Y = \emptyset$, we have a partial conflict between the experts. If the experts are reliable then, we can transfer the mass on X, Y or $X \cup Y$, such as the $DPCR$ proposed in [8]. If the experts are not sure then a part of the mass can also be transfered on the complement of X and Y.

Hence we propose the function w given in the Table 5.1 if $X \cap Y = \emptyset$, and in the Table 5.2 if $X \cap Y \neq \emptyset$. The given weights have to be normalized by a factor noted N.

When $X \cap Y = \emptyset$	
Element	Weight N
X	$\alpha_{1X} m_1(X)$
Y	$\alpha_{2Y} m_2(Y)$
$c(X)$	$(1 - \alpha_{1X})(1 - m_1(X))$
$c(Y)$	$(1 - \alpha_{2Y})(1 - m_2(Y))$
$X \cup Y$	$(1 - \alpha_{1X}\alpha_{2Y})(1 - m_1(X)m_2(Y))$
$c(X) \cap c(Y) \neq \emptyset$	$(1 - \alpha_{1X})(1 - \alpha_{2Y})(1 - m_1(X))(1 - m_2(Y))$
$c(X) \cup c(Y) = \Theta$	$(1 - (1 - \alpha_{1X})(1 - \alpha_{2Y}))(1 - (1 - m_1(X))(1 - m_2(Y)))$

Table 5.1: Weighting function w when $X \cap Y = \emptyset$.

When $X \cap Y \neq \emptyset$	
Element	weight N
$X \cap Y$	$\alpha_{1X}\alpha_{2Y} m_1(X)m_2(Y)$
$X \cup Y$	$(1 - \alpha_{1X}\alpha_{2Y})(1 - m_1(X)m_2(Y))$

Table 5.2: Weighting function w when $X \cap Y \neq \emptyset$.

In this form, if the expert 1, for example, is not reliable, we do not transfer on $c(X)$. So we propose the function w is given by the Table 5.3, still for $X \cap Y \neq \emptyset$. In this case, the rule will have a behavior nearer than the average than the conjunctive because the weights on X and Y are higher than the weight on $X \cap Y$. So, we can also propose the function w as in Table 5.4 in order to avoid that.

When $X \cap Y \neq \emptyset$	
Element	weight N
$X \cap Y$	$\alpha_{1X}\alpha_{2Y}m_1(X)m_2(Y)$
$X \cup Y$	$(1-\alpha_{1X}\alpha_{2Y})(1-m_1(X)m_2(Y))$
X	$\alpha_{1X}m_1(X)$
Y	$\alpha_{2Y}m_2(Y)$
$c(X) \neq \emptyset$	$(1-\alpha_{1X})(1-m_1(X))$
$c(Y) \neq \emptyset$	$(1-\alpha_{2Y})(1-m_2(Y))$
$c(X) \cap c(Y) \neq \emptyset$	$(1-\alpha_{1X})(1-\alpha_{2Y})(1-m_1(X))(1-m_2(Y))$
$c(X) \cup c(Y)$	$(1-(1-\alpha_{1X})(1-\alpha_{2Y}))(1-(1-m_1(X))(1-m_2(Y)))$
$X \cup c(Y)$	$(1-\alpha_{1X}(1-\alpha_{2Y}))(1-m_1(X)(1-m_2(Y)))$
$c(X) \cup Y$	$(1-(1-\alpha_{1X})\alpha_{2Y})(1-(1-m_1(X))m_2(Y))$
$X \cap c(Y) \neq \emptyset$	$\alpha_{1X}(1-\alpha_{2Y})m_1(X)(1-m_2(Y))$
$c(X) \cap Y \neq \emptyset$	$(1-\alpha_{1X})\alpha_{2Y}(1-m_1(X))m_2(Y)$

Table 5.3: Weighting function w when $X \cap Y \neq \emptyset$.

When $X \cap Y \neq \emptyset$	
Element	weight N
$X \cap Y$	$\alpha_{1X}\alpha_{2Y}m_1(X)m_2(Y)$
$X \cup Y$	$(1-\alpha_{1X}\alpha_{2Y})(1-m_1(X)m_2(Y))$
X	$(\alpha_{1X}m_1(X))^2$
Y	$(\alpha_{2Y}m_2(Y))^2$
$c(X) \neq \emptyset$	$((1-\alpha_{1X})(1-m_1(X)))^2$
$c(Y) \neq \emptyset$	$((1-\alpha_{2Y})(1-m_2(Y)))^2$
$c(X) \cap c(Y) \neq \emptyset$	$(1-\alpha_{1X})(1-\alpha_{2Y})(1-m_1(X))(1-m_2(Y))$
$c(X) \cup c(Y)$	$(1-(1-\alpha_{1X})(1-\alpha_{2Y}))(1-(1-m_1(X))(1-m_2(Y)))$
$X \cup c(Y)$	$(1-\alpha_{1X}(1-\alpha_{2Y}))(1-m_1(X)(1-m_2(Y)))$
$c(X) \cup Y$	$(1-(1-\alpha_{1X})\alpha_{2Y})(1-(1-m_1(X))m_2(Y))$
$X \cap c(Y) \neq \emptyset$	$\alpha_{1X}(1-\alpha_{2Y})m_1(X)(1-m_2(Y))$
$c(X) \cap Y \neq \emptyset$	$(1-\alpha_{1X})\alpha_{2Y}(1-m_1(X))m_2(Y)$

Table 5.4: Weighting function w when $X \cap Y \neq \emptyset$.

5.6.2 Some examples

- if $X \cap Y = \emptyset$ and $\alpha_{1X} = \alpha_{2Y} = 1$, then the only weights are $m_1(X)$ and $m_2(Y)$ respectively on X and Y.

- if $X \cap Y = \emptyset$ and $\alpha_{1X} = 1$ and $\alpha_{2Y} = 0$, then the only weights are $m_1(X)$, $(1-m_2(Y))$, $m_1(X)(1-m_2(Y))$ and $1-m_1(X)m_2(Y)$ respectively on X, $c(Y)$, $X \cap c(Y)$ and $X \cup Y$.

- if $X \cap Y \neq \emptyset$ and $\alpha_{1X} = \alpha_{2Y} = 1$, then the only weights are $m_1(X)m_2(Y)$, $m_1(X)$ (or $m_1(X)^2$) and $m_2(Y)$ (or $m_2(Y)^2$) respectively on $X \cap Y$, X and Y.

- if $X \cap Y \neq \emptyset$ and $\alpha_{1X} = 1$ and $\alpha_{2Y} = 0$, then the only weights are $m_1(X)$, $(1-m_2(Y))$, $m_1(X)(1-m_2(Y))$ and $1-m_1(X)m_2(Y)$ respectively on X, $c(Y)$, $X \cap c(Y)$ and $X \cup Y$.

- if $X \cap Y = \emptyset$ and $m_1(X)m_2(Y) = 1$, then the only weights are α_{1X} and α_{2Y} respectively on X and Y.

- if $X \cap Y \neq \emptyset$ and $m_1(X)m_2(Y) = 1$, then the only weights are $\alpha_{1X}\alpha_{2Y}$, α_{1X} and α_{2Y} respectively on $X \cap Y$, X and Y.

5.6.3 The fusion of $s \geq 2$ experts including their reliability

We note $Y_1, \cdots Y_s$ the responses of the experts. The function w is then given by the Table 5.5 if $\cap_{j=1}^{s} Y_j = \emptyset$:

$\cap_{j=1}^{s} Y_j = \emptyset$	
Element	Weight N
Y_j	$\alpha_{jY_j} m_j(Y_j)$
$c(Y_j)$	$(1-\alpha_{jY_j})(1-m_j(Y_j))$
$\cup_{j_1=1}^{n_1} Y_{j_1} \bigcup \cup_{j_2=1}^{n_2} c(Y_{j_2})$ with $n_1+n_2=s$	$\left(1-\prod_{j_1=1}^{n_1} \alpha_{j_1 Y_{j_1}} \prod_{j_2=1}^{n_2} (1-\alpha_{j_2 Y_{j_2}})\right) \times \left(1-\prod_{j_1=1}^{n_1} m_{j_1}(Y_{j_1}) \prod_{j_2=1}^{n_2} (1-m_{j_2}(Y_{j_2}))\right)$
$\cup_{j=1}^{s} Y_j$	$(1-\prod_{j=1}^{s} \alpha_{jY_j})(1-\prod_{j=1}^{s} m_j(Y_j))$
$\cap_{j_1=1}^{n_1} Y_{j_1} \bigcap \cap_{j_2=1}^{n_2} c(Y_{j_2})$ if $\neq \emptyset$, with $n_1+n_2=s$	$\prod_{j_1=1}^{n_1} \alpha_{j_1 Y_{j_1}} m_{j_1}(Y_{j_1}) \times \prod_{j_2=1}^{n_2} (1-\alpha_{j_2 Y_{j_2}})(1-m_{j_2}(Y_{j_2}))$
$\cap_{j=1}^{s} c(Y_j)$ if $\neq \emptyset$	$\prod_{j=1}^{s} (1-\alpha_{jY_j})(1-m_j(Y_j))$
$\cup_{j=1}^{s} c(Y_j)$	$\left(1-\prod_{j=1}^{s} (1-\alpha_{jY_j})\right) \times \left(1-\prod_{j=1}^{s} (1-m_j(Y_j))\right)$

Table 5.5: Weighting function w when $\cap_{j=1}^{s} Y_j = \emptyset$.

The function w given in Table 5.6 if $\cap_{j=1}^{s} Y_j \neq \emptyset$:

$\cap_{j=1}^{s} Y_j \neq \emptyset$	
Element	weight N
$\cap_{j=1}^{s} Y_j$	$\prod_{j=1}^{s} \alpha_{jY_j} m_j(Y_j))$
$\cup_{j=1}^{s} Y_j$	$(1 - \prod_{j=1}^{s} \alpha_{jY_j})(1 - \prod_{j=1}^{s} m_j(Y_j))$
Y_j	$\alpha_{jY_j} m_j(Y_j)$
$c(Y_j)$ if $\neq \emptyset$	$(1 - \alpha_{jY_j})(1 - m_j(Y_j))$
$\cup_{j_1=1}^{n_1} Y_{j_1} \bigcup \cup_{j_2=1}^{n_2} c(Y_{j_2})$ if $\neq \emptyset$, with $n_1 + n_2 = s$	$\left(1 - \prod_{j_1=1}^{n_1} \alpha_{j_1 Y_{j_1}} \prod_{j_2=1}^{n_2} (1 - \alpha_{j_2 Y_{j_2}})\right) \times \left(1 - \prod_{j_1=1}^{n_1} m_{j_1}(Y_{j_1}) \prod_{j_2=1}^{n_2} (1 - m_{j_2}(Y_{j_2}))\right)$
$\cap_{j_1=1}^{n_1} Y_{j_1} \bigcap \cap_{j_2=1}^{n_2} c(Y_{j_2})$ if $\neq \emptyset$, with $n_1 + n_2 = s$	$\prod_{j_1=1}^{n_1} \alpha_{j_1 Y_{j_1}} m_{j_1}(Y_{j_1}) \times \prod_{j_2=1}^{n_2} (1 - \alpha_{j_2 Y_{j_2}})(1 - m_{j_2}(Y_{j_2}))$
$\cap_{j=1}^{s} c(Y_j)$ if $\neq \emptyset$	$\prod_{j=1}^{s} (1 - \alpha_{jY_j})(1 - m_j(Y_j))$
$\cup_{j=1}^{s} c(Y_j)$	$\left(1 - \prod_{j=1}^{s} (1 - \alpha_{jY_j})\right) \times \left(1 - \prod_{j=1}^{s} (1 - m_j(Y_j))\right)$

Table 5.6: Weighting function w when $\cap_{j=1}^{s} Y_j \neq \emptyset$.

Note that with extension $T(\mathbf{Y}) \neq D^{\{\mathbf{Y}, c(\mathbf{Y})\}} \setminus \{\emptyset\}$, but $T(\mathbf{Y}) \subset D^{\{\mathbf{Y}, c(\mathbf{Y})\}} \setminus \{\emptyset\}$.

5.7 Conclusions

We have constructed a Redistribution to Subsets or Complements (RSC) class of fusion rules and we gave eight particular examples. All RSC rules work on the fusion spaces S^{Θ} and 2^{Θ}. But the RSC rules involving complements do not work on the hyper-power set D^{Θ}.

In order to choose what particular RSC rule to apply we need to take into consideration the user's feasability, confidence/non-confidence in some hypotheses, more or less prudence of the user, optimistic/pessimistic redistribution, etc. In general, if $X \cap Y = \emptyset$, the mass of $X \cap Y$ is transferred either to $c(X \cap Y)$, or to subsets of $c(X \cap Y)$, or to X and Y, or to subsets of $X \cup Y$ proportionally with respect to the masses, or cardinals, or both masses and cardinals, or other parameters of the elements that receive redistributed masses. We can even transfer the mass of $X \cap Y$ when $X \cap Y \neq \emptyset$ in the same way as aforementioned; the transfer of $m(X \cap Y)$ when $X \cap Y \neq \emptyset$ is done or not depending on the confidence/non-confidence of the user in the set $X \cap Y$. A more general theoretical extension of these RSC rules is presented at the end of this chapter. Those can generate new classes of fusion rules.

5.8 References

[1] M. Daniel, *Comparison between DSm and MinC combination rules*, Chap. 10, pp. 223-241, in [10].

[2] M. Daniel, *Classical Combination Rules Generalized to DSm Hyper-power Sets and their Comparison with the Hybrid DSm Rule*, Chap. 3, pp. 89-112, in [13].

[3] J. Dezert, A. Martin, F. Smarandache,*Comments on "A new combination of evidence based on compromise" by K. Yamada*, Fuzzy Sets and Systems, Vol. 160, Issue 6, pp. 853–855, March 2009.

[4] D. Dubois, H. Prade, *A set-theoretic view of belief functions - Logical operations and approximation by fuzzy sets*, International journal of General Systems, Vol. 12, No. 3, pp. 193-226, 1986.

[5] D. Dubois, H. Prade, *Representation and combination of uncertainty with belief functions and possibility measures*, Computational Intelligence, Vol. 4, pp. 244-264, 1988.

[6] E. Lefevre, O. Colot, P. Vannoorenberghe, *Belief functions combination and conflict management*, Information Fusion Journal, Elsevier Publisher, Vol. 3, No. 2, pp. 149–162, 2002.

[7] A. Martin, C. Osswald, *A new generalization of the proportional conflict redistribution rule stable in terms of decision*, Chap. 2, pp. 69-88, in [13].

[8] A. Martin and C. Osswald, ,*Toward a combination rule to deal with partial conflict and specificity in belief functions theory*, International Conference on Information Fusion, Québec, Canada, 9-12 July 2007.

[9] A. Martin, C. Osswald, J. Dezert, F. Smarandache,*General combination rules for qualitative and quantitative beliefs*, to appear in Journal of Advances in Information Fusion, 2009.

[10] F. Smarandache, J. Dezert (Editors), *Applications and Advances of DSmT for Information Fusion*, Vol. 1, American Research Press, Rehoboth, August 2004. `http://www.gallup.unm.edu/~smarandache/DSmT-book1.pdf`.

[11] F. Smarandache, J. Dezert, *Combination of beliefs on hybrid DSm models*, Chap. 4, pp. 61-103, in [10].

[12] F. Smarandache, J. Dezert, *Information Fusion Based on New Proportional Conflict Redistribution Rules*, International Conference on Information Fusion, Philadelphia, U.S.A., July 2005.

[13] F. Smarandache, J. Dezert (Editors), *Applications and Advances of DSmT for Information Fusion*, Vol. 2, American Research Press, Rehoboth, August 2006. `http://www.gallup.unm.edu/~smarandache/DSmT-book2.pdf`.

[14] F. Smarandache, J. Dezert, *Proportional Conflict Redistribution Rules for Information Fusion*, Chap. 1, pp. 3-68, in [13].

[15] K. Yamada, *On new combination of evidence by compromise*, In proc. of Joint 3rd International Conference on Soft Computing and Intelligent Systems and 7th International Symposium on advanced Intelligent Systems, Tokyo, Japan, 20-24 september, 2006.

[16] K. Yamada, *A new combination of evidence based on compromise*, Fuzzy Sets and Systems, Vol. 159, No. 13, pp. 1689–1708, july 2008.