CHARACTERIZATION OF EM SEA CLUTTER WITH α-STABLE DISTRIBUTION

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ABSTRACT

In this contribution, an accurate description of the ocean backscatter from a probability density function is proposed. The Elfouhaily spectrum has been used to generate a realistic sea surface. The scattering field will be computed by using the Physical Optics (PO). The K distribution has been already used to characterize the Radar Cross Section (RCS) of the sea surface. However, the probability density function of the RCS can have heavy tails. Consequently, we use the α-stable distributions which can take care the property of heavy tails. The probability density function is estimated with a least squared method. We finally compare the results obtained with each model by using the Kolmogorov-Smirnov test from several random surfaces and a statistical study is made by giving a boxplot of the estimated parameters of the α-stable distribution.

Index Terms— Elfouhaily spectrum, Physicals Optics, RCS, α-stable distribution.

1. INTRODUCTION

An accurate description of the ocean backscatter from a probability density function is essential in a problem of maritime radar target detection. This is usually due to the fact that the sea clutter are highly non-stationnary.

The probability density function of the RCS has been mainly studied with the K distribution. In [1], the authors proved that the K distribution fits sea clutter data well. The variation of the mean and the shape parameter of the K distribution have been already studied by varying the polarisation and the geometrical aspects [2, 3, 4]. The probability density function of the RCS can have a heavy tail and can be asymmetric. A probability density function is said to have a heavy tail if the tail of the distribution decreases more slowly that the tail of a Gaussian probability density function. A probability density function is said asymmetric if it is not possible to find a mode such as the probability density function is symmetric. These properties can be modeled by the α-stable distributions. Consequently, we study for the rest of the paper the RCS with the α-stable distribution.

The α-stable distribution have been mainly used to model clutter in the imagery domain such as in [5, 6]. In [7], the author proposed to characterize the RCS of the sea clutter and a ship in a backscattering configuration. In this paper, we extend the study in a bistatic configuration.

The paper layout is as follows. In section 2, we explain how we generate the sea surface and the way to compute the scattering field. In section 3, we introduce the α-stable distributions and the method to fit the α-stable distribution. In section 4, the RCS of several random surfaces are computed with the Physical Optics (PO). A bar chart is built from the RCS and is fitted with the α-stable distribution. The quality of model is evaluated by a Kolmogorov-Smirnov test. A boxplot of the the parameters of the α-stable distributions is finally given.

2. SEA CLUTTER RCS

In this section, the electomagnetic characteristics of the sea surface are modeled by the Debye model and the geometrical aspects are modeled by the Elfouhaily spectrum. We finally compute the RCS of the sea surface by using the Physical Optics.

2.1. Debye model and Elfouhaily spectrum

The Debye model [8] is used to model the relative permittivity of the sea surface. The relative permittivity ϵ_r is function of the temperature T and the salinity S:

$$\epsilon_r = \epsilon_{r,\infty} + \frac{\epsilon_s - \epsilon_{r,\infty}}{1 + jw\tau_r} - j\frac{\sigma_s}{w\epsilon_0} \tag{1}$$

where w is the radian frequency, ϵ_0 the vacuum permittivity, $\epsilon_{r,\infty}$ the dielectric constant at infinite frequency and τ_r the relaxation time.

The wave height of the sea can be modeled by a random function of the position $\zeta(x, y)$. The sea surface is described as a linear superposition of individual sinusoidal waves. A spectral representation of wave $\mathcal{S}$ have been proposed:

$$\mathcal{S}(\mathbf{K}, \psi) = \frac{1}{|\mathbf{K}|} S_{omni}(|\mathbf{K}|) S_{spread}(\psi) \tag{2}$$

with $\mathbf{K}$ the wavenumber vector, S_{omni} is the omnidirectional wave height spectrum and S_{spread} is the spreading function with the direction of wind ψ.

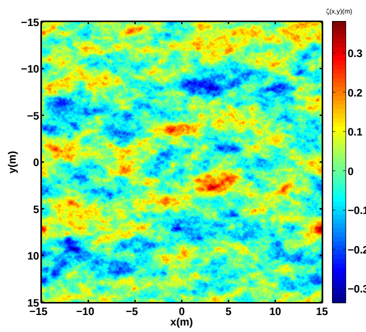

Fig. 1. Generated sea surface of dimension $30\mathrm{m} \times 30\mathrm{m}$ with the Elfouhaily spectrum.

We choose to work with the Elfouhaily spectrum [9]. Indeed, this spectrum can take care the fetch, the gravity and the capillarity waves. We give an example of a sea surface with the temperature $T = 20°$, the salinity $S_{al} = 35ppt$, wind-speed $V = 3\mathrm{m/s}$ and direction of wind $\psi = 0°$ in Figure 1.

In the next subsection, we explain the way to compute the RCS of sea surface with the Physical Optics.

2.2. Physical Optics

The scattering field is estimated by using the Physical Optics (PO) [10]. This method supposed an electromagnetic wave $(\mathbf{E}_i, \mathbf{H}_i)$ which produced surface currents on a target. The electromagnetic wave creates the induced magnetic $\mathbf{J}_m$ and electric $\mathbf{J}_e$ currents given by:

$$\mathbf{J}_m = -\mathbf{n} \times \mathbf{E} \quad \mathbf{J}_e = \mathbf{n} \times \mathbf{H} \tag{3}$$

where n is the unit normal vector to the surface, $\mathbf{E}$ and $\mathbf{H}$ are respectively the total electric and magnetic fields at the surface. The incident field can be considered as a plane wave

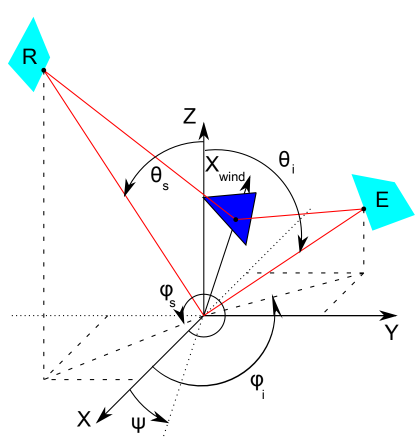

Fig. 2. Geometrical configuration.

if the source illuminating the target is at a far enough distance. The scattering field from the illuminated surface S is given by:

$$\mathbf{E}_s = \frac{\mathbf{i}ke^{(-\mathbf{i}kR)}}{4\pi R} \int_S [\mathbf{k}_s \times (\eta \mathbf{k}_s \times \mathbf{J}_e + \mathbf{J}_m)] e^{-\mathbf{i}k\mathbf{k}_s \cdot \mathbf{r}} ds \tag{4}$$

where k is the wavenumber, R is the distance between the center of the referential and the receiver, $\mathbf{k}_i$ and $\mathbf{k}_s$ are respectively the unit directional vectors of the incident and scattering electromagnetic wave. The parameter η represents the impedance of the medium and $\mathbf{r}$ is the position vector of a point in S. The equation (4) is complicated but it can be solved by decomposing the sea surface into triangular sub-regions [11].

3. THE α-STABLE DISTRIBUTION

All the definitions use in this section are defined in [12].

3.1. Definition of stability

A random variable X is stable if for all $(a,b) \in \mathbb{R}^+ \times \mathbb{R}^+$, there are $c \in \mathbb{R}^+$ and $d \in \mathbb{R}$ such that:

$$aX_1 + bX_2 = cX + d \tag{5}$$

with X_1 and X_2 two independent α-stable random variables which follow the same distribution as X. If Equation (5) defines the notion of stability, it does not give any indication as to how to parametrize an α-stable distribution. We therefore prefer to use the definition given by characteristic function to refer to an α-stable distribution.

3.2. Probability density function

Several equivalent definitions have been suggested in the literature to parametrize an α-stable distribution from its characteristic function [13, 14]. Zolotarev [14] proposed the following:

$$\phi_{S_\alpha(\beta,\gamma,\delta)}(t) = \begin{cases} e^{(it\delta - |\gamma t|^\alpha [1 + i\beta \tan(\frac{\pi\alpha}{2}) s(t)(|t|^{1-\alpha} - 1)])} & \text{if } \alpha \neq 1 \\ e^{(it\delta - |\gamma t| [1 + i\beta \frac{2}{\pi} s(t) \log|t|])} & \text{if } \alpha = 1 \end{cases} \tag{6}$$

with $\alpha \in]0,2]$ the characteristic exponent, $\beta \in [-1,1]$ the skewness parameter, $\gamma \in \mathbb{R}^{+*}$ the scale parameter and $\delta \in \mathbb{R}$ is the location parameter. The representation of an α-stable probability density function, noted $f_{S_\alpha(\beta,\gamma,\delta)}$, is obtained by calculating the Fourier transform of its characteristic function:

$$f_{S_\alpha(\beta,\gamma,\delta)}(x) = \int_{-\infty}^{\infty} \phi_{S_\alpha(\beta,\gamma,\delta)}(t) e^{(-itx)} dt \tag{7}$$

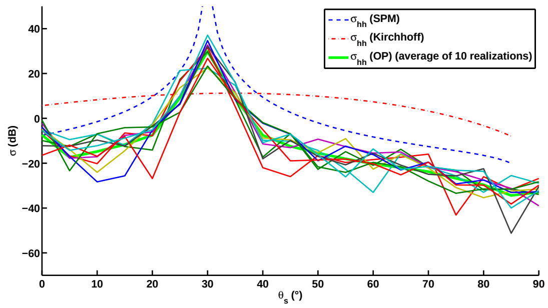

Fig. 3. Bistatic RCS of a random sea surface generated 10 times for a hh-ploarization with $\theta_i = 30°$.

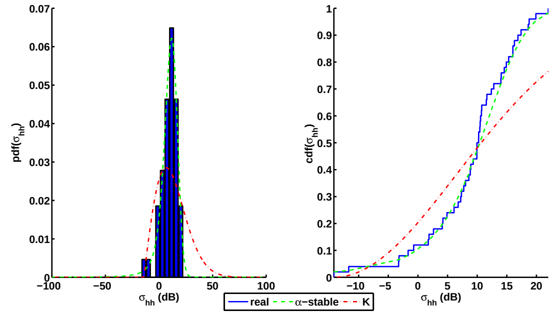

Fig. 4. An example of RCS and its estimations with $\theta_i = 30°$ and $\theta_s = 25°$.

3.3. Estimation

The estimators of α-stable distributions are decomposed into four families: the sample quantile methods [15], the sample characteristic methods [16] and the Maximum Likelihood estimation [17]. We use a Least Squared estimation to fit the RCS because this method minimizes the sum of squared residuals.

4. SIMULATION AND RESULTS

4.1. Goodness-of-fit test

We generated the RCS of 500 sea surfaces with (30×30) m-sized with: the wind speed $V = 3\text{m/s}$, the wind direction $\psi = 0°$, the temperature $T = 20°$, the salinity $S_{al} = 35ppt$, θ_i and $\theta_s \in [0°; 90°]$. The frequency f is equal to 10 GHz and we work in co-polarization Horizontal-Horizontal. The PO results seems to show good accuracy with results obtained by the Small Perturbations Method [18] (Figure 3). For our statistical study, we fix the parameter $\theta_s = 45°$ and build and histogram from 50 RCS. We fit the histogram with the α-stable model. This step is realized 15 times. The same study is realized by fixing $\theta_i \in [0°; 90°]$ and by considering θ_s as a random variable.

The quality of the α-stable distribution is evaluated by a Kolmogorov-Smirnov goodness-of-fit test, which compare the data cumulative distribution function with the cumulative distribution function of the fitted α-stable distribution. The significance level is set at $l = 0.05$.

We compare the α-stable model with the K distribution. A random variable X is said to be a K distribution with parameters $\nu > 1$ and $a \in \mathbb{R}^{+*}$, noted $X \sim \mathcal{K}(\nu, a)$, if its probability density function has the form:

$$f_{\mathcal{K}(a,\nu)}(x) = \begin{cases} \dfrac{2}{a\Gamma(\nu+1)} \left(\dfrac{x}{2a}\right)^{\nu+1} K_\nu\left(\dfrac{x}{a}\right) & \text{if } x \geq 0 \\ 0 & \text{otherwise.} \end{cases} \quad (8)$$

where a is the scaling parameter, ν is the shape parameter, $\Gamma(.)$ the gamma function and K_ν is the modified Bessel function of the second kind of order ν [19].

The % of success rate for the Kolmogorov-Smirnov test is 97.5 for the α-stable distribution and 9.55 for the K distribution. We give an example of estimation. We can observe in Figure 4 that the α-stable distribution gives a better fit than the K distribution.

4.2. Boxplot

The boxplot is a convenient way of graphically depicting groups of numerical data: the smallest observation, the first quartile, the median, the third quartile, the largest observation and the outlier. We give the boxplot for the configuration: the wind speed $V = 3\text{m/s}$, the wind direction $\psi = 0°$, the temperature $T = 20°$, the salinity $S_{al} = 35ppt$, $\theta_s = 25°$ and $\theta_i \in [0°; 90°]$. We can observe that the median for the parameter β is equal to 1. The parameter of position δ function of θ_i describe the shape of the RCS. For the parameter α and γ, it is difficult to extract information: we can just say that $\gamma \in [2, 5.5]$.

By varying the θ_i and θ_s between $[0°, 90°]$, we generate the probability density function of parameter δ (Figure 6). We can observe that there are three modes: the smallest mode corresponds to $\theta_i = \theta_s$ (specular direction).

5. CONCLUSION

In this paper, we propose to characterize the probability density function of RCS with the α-stable distributions. The α-stable distributions give better fit than the K distribution. The parameter δ gives more information than the other parameters. In future works, we try to characterize the sea clutter RCS by varying the polarization, the wind speed and the direction of speed. It will be interesting to compare the results with a real dataset.

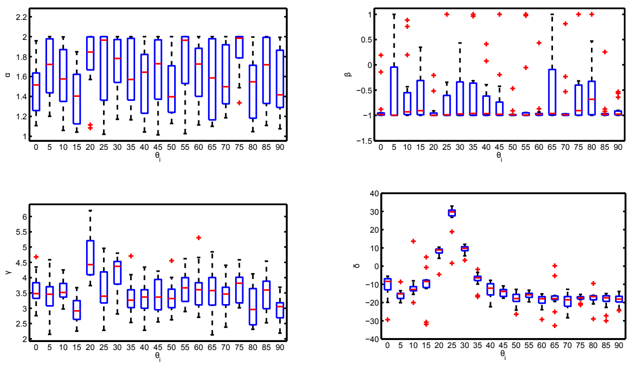

Fig. 5. Boxplot of the α-stable parameter with $\theta_s = 25°$.

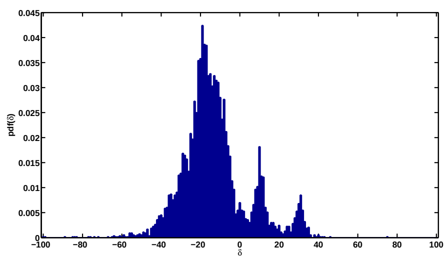

Fig. 6. Probability density function of parameter δ.

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