A Distance Between Continuous Belief Functions

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Abstract. *In the theory of belief functions, distances between basic belief assignments are very important in many applications like clustering, conflict measuring, reliability estimation. In the discrete domain, many measures have been proposed, however, distance between continuous belief functions have been marginalized due to the nature of these functions. In this paper, we propose an adaptation inspired from the Jousselme's distance for continuous belief functions.*

Keywords: Theory of belief functions, continuous belief functions, distance, scalar product

1 Introduction

The theory of belief function (also referred to as the mathematical theory of evidence) is one of the most popular quantitative approach. It is known for its ability to represent uncertain and imprecise information.

It is a strong formalism widely used in many research areas: medical, image processing,... It has been used thanks to its ability to manage imperfect information.

In the discrete case, information fusion, has known a large success, focusing on the study of conflict between belief functions. Among these researches, it has been considered that a distance between two bodies of evidence can be interpreted as a conflict measure used during combination as presented in [7]. Smets, in [15], extended the theory of belief functions on real numbers thinking over continuous belief functions by presenting a complete description.
In the discrete case, distances between probability distributions can be considered as a definition of dissimilarity in the theory of belief functions. Ristic and Smets in [10], defined a distance based on Dempster's conflict factor, others proposed geometrical ones like Jousselme *et al.* in [5].

In this paper we are interested to adapt the notion of distance to the continuous belief functions, and study the behavior according to two different types of distributions.

This paper is organized as follow: in Section 2 we recall some basic concepts of the theory of belief functions in the discrete case, and present the notion of distance. Therefore, in Section 3, we introduce the continuous belief functions, some of their properties and characterization. After this, we propose in Section 4 an adaptation of the Jousselem's distance using Smet's formalism. Finally, in Section 5, the proposed distance is used to measure the distance to the continuous belief functions where the probability density functions are following in the first place all the normal then the exponential distribution, and the contribution of using this distance instead of a classical scalar product.

2 Belief function theory background

This section recalls the necessary background notions related to the theory of belief functions. It has been developed by Dempster in his work on upper and lower probabilities [1]. Afterwards, it was formalized in a mathematical framework by Shafer in [11]. This theory is able to deal and represent imperfect (uncertain, imprecise and /or incomplete) information.

2.1 Discrete belief functions

Let us consider a variable x taking values in a finite set $\Omega = \{\omega_1, \cdots, \omega_n\}$ called *the frame of discernment.*
A *basic belief assignment* (*bba*) is defined on the set of all subsets of Ω, named power set and noted 2^Ω. It affects a real value from $[0, 1]$ to every subset of 2^Ω reflecting sources amount of belief on this subset. A bba m verifies:

$$\sum_{X \subseteq \Omega} m(X) = 1. \tag{1}$$

Given a bba m we can associate some other functions defined as:

- Credibility: measures the strength of the evidence in favor of a set of propositions for all $X \in 2^\Omega \setminus \emptyset$:
$$bel(X) = \sum_{Y \subseteq X, Y \neq \emptyset} m(Y). \tag{2}$$
- Plausibility: quantifies the maximum amount of belief that could be given to a X of the frame of discernment for all $X \in 2^\Omega \setminus \emptyset$:
$$pl(X) = \sum_{Y \in 2^\Omega, Y \cap X \neq \emptyset} m(Y). \tag{3}$$

- Commonality: measures the set of *bbas* affected to the focal elements included in the studied set, for all $X \in 2^{\Omega}$:

$$q(X) = \sum_{Y \supseteq X} m(Y). \tag{4}$$

- Pignistic probability transformation: proposed in[12], transforms a *bba* m into a probability measure for all $X \in 2^{\Omega}$:

$$betP(X) = \sum_{Y \neq \emptyset} \frac{\mid X \cap Y \mid}{\mid Y \mid} \frac{m(Y)}{1 - m(\emptyset)}. \tag{5}$$

The bba (m) and the functions *bel*, *pl* are different expressions of the same information.

2.2 Combination rules

In the belief function theory, Dempster in [1] proposed the first combination rule. It is defined for two bbas m_1, m_2, $\forall X \in 2^{\Omega}$ with $X \neq \emptyset$ by:

$$m_{DS}(X) = \frac{1}{1-k} \sum_{A \cap B = X} m_1(A) m_2(B), \tag{6}$$

where k is generally called the global conflict of the combination or its inconsistency, defined by $k = \sum_{A \cap B = \emptyset} m_1(A) m_2(B)$ and $1 - k$ is a normalization constant.

The Dempster combination rule is not the only one used to combine, Dubois and Prade proposed a disjunctive one. Smets [14] proposed to consider an open world, therefore the conjunctive rule is a non-normalized one and for two basic belief assignments m_1, m_2 for all $X \in 2^{\Omega}$ by:

$$m_{conj}(X) = \sum_{A \cap B = X} m_1(A) m_2(B) := (m_1 \oplus m_2)(X). \tag{7}$$

and $k = m_{conj}(\emptyset)$ is considered as a non expected solution.

This combination rule is very useful because, in one hand it decreases the vagueness and, on the other hand, it increases the belief of the observed focal elements.

3 A distance between two discrete belief functions

The aim of this paper is to define a distance between continuous belief functions, let us begin by introducing some basic concepts relative to distances.

3.1 Properties of the distance

The distance defined between two elements A and B in a set I satisfies the following requirement:

- Nonnegativity: $d(A, B) \geq 0$.
- Nondegeneracy: $d(A, B) = 0 \Leftrightarrow A = B$.
- Symmetry: $d(A, B) = d(B, A)$.
- Triangle inequality: $d(A, B) \leq d(A, C) + d(C, B)$.

3.2 Scalar product

On a defined set, a distance can be measured with a scalar product between two vectors f and g. A dot product has a symmetric and a bilinar form, and is defined positive.

- $\langle f, g\rangle = \langle g, f\rangle$
- $\langle f, f\rangle > 0$
- $\langle f, f\rangle = 0 \Rightarrow f = \overrightarrow{0}$: the zero vector.

We have $\langle f, f\rangle = \|f\|^2$ which is the square norm of f.

To measure a distance using a scalar product, we use the following expression:

$$d_{SP}(f, g) = \sqrt{\frac{1}{2}(\|f\|^2 + \|g\|^2 - 2\langle f, g\rangle))} \tag{8}$$

In the next section, we will present a particular measure based on a scalar product able to define a distance between two discrete belief functions in the theory of belief functions.

3.3 A distance between two discrete belief functions

The distance introduced in [5] is the most appropriate distance to measure the dissimilarity between two *bbas* m_1, m_2 according to [6] after making a comparison of distances in belief functions theory. for two *bbas*: m_1, m_2 on 2^Ω:

$$d(m_1, m_2) = \sqrt{\frac{1}{2}(\|m_1\|^2 + \|m_2\|^2 - 2\langle m_1, m_2\rangle))} \tag{9}$$

and $\langle m_1, m_2\rangle$ is the scalar product defined by:

$$\langle m_1, m_2\rangle = \sum_{i=1}^{n}\sum_{j=1}^{n} m_1(A_i)m_2(A_j)\frac{|A_i \cap A_j|}{|A_i \cup A_{j|}} \tag{10}$$

where $n = |2^\Omega|$.

Therefore, $d(m_1, m_2)$ is considered as an illustration of the scalar product where the factor $\frac{1}{2}$ is needed to normalize d and guarantee that $0 \leq d(m_1, m_2) \leq 1$.

According to [6], this metric distance respects all the properties expected by a distance and it can be considered as an appropriate measure of the difference or the lack of similarity between two *bbas*.
This distance is based on the dissimilarity of Jaccard defined as:

$$Jac(A, B) = \frac{|A \cap B|}{|A \cup B|}. \tag{11}$$

Moreover, $d(m_1, m_2)$ can be called a total conflict measure, which is an interesting property to compute the total conflict based on a measurable distance like presented in [7].

4 Continuous belief functions

Strat in [17], and Smets in [15] proposed a definition of continuous belief functions. Strat, proposed to represent a mass function of a discretized variable in a triangular matrix. This representation reduces the number of the propositions and simplifies the computation of the intersections by focusing only on the contiguous intervals.
Smets used the same representation and proposed the belief functions in the extended set of reals noted $\overline{\mathbb{R}} = \mathbb{R} \cup \{-\infty, +\infty\}$.
However, using the belief function framework to model information in a continuous frame is not an easy task due to the complex nature of the focal elements. Comparing to the discrete domain, on real numbers, in [15] *bba* becomes *basic belief densities* (*bbd*) defined on an interval $[a, b]$ of $\mathbb{R}$.

4.1 Belief functions on $\overline{\mathbb{R}}$

Smets generalized the classical *bba* into a basic belief density (*bbd*) noted m^I on the interval I. In the definition of the *bbd*, all focal elements are closed intervals or $\emptyset$. Given a normalized *bbd* m^I, he defined an other function f on $\mathbb{R}^2$, where $f(a, b) = m^I([a, b)]$ for $a \leq b$ and $f(a, b) = 0$ whenever $a > b$.
f is called a probability density function (*pdf*) on $\mathbb{R}^2$.

- Credibility:

$$bel^{\overline{\mathbb{R}}}([a, b]) = \int_{x=a}^{x=b} \int_{y=x}^{y=b} m^{\overline{\mathbb{R}}}([x, y])dydx. \tag{12}$$

- Plausibility:

$$pl^{\overline{\mathbb{R}}}([a, b]) = \int_{x=-\infty}^{x=b} \int_{y=max(a,x)}^{y=+\infty} m^{\overline{\mathbb{R}}}([x, y])dydx. \tag{13}$$

– Commonality:

$$q^{\overline{\mathbb{R}}}([a,b]) = \int_{x=-\infty}^{x=a} \int_{y=b}^{y=+\infty} m^{\overline{\mathbb{R}}}([x,y])dydx. \qquad (14)$$

Smets' approach is based on the description of focal elements from a continuous function, where the frame of discernment is built having on connected sets of $\overline{\mathbb{R}}$.

Nguyen introduced in [8] the notions of a source constituted by a probability space and a multivalued mapping Γ able to define the lower probability. According to him, the probability distribution of a random set Γ is the basic belief probability assignment. He shows the existence of a correspondence between belief function in a source and the probability distribution of a random set. This correspondence is established by constructing a probability distribution, which its density is already defined.

Doré *et al.* in [2] proposed a similar approach founded on an index function that can be assumed as Γ. This function can describe the set of focal elements of a continuous belief function. In this case, every index has its own probability measure where there is an allocated weight to a set of focal elements using a credal measure.

4.2 Belief functions associated to a probability density

A probability density function is an expression of an expert's belief. This probability can be expressed according to a basic belief density which is described using a normal (Gaussian), exponential distribution.
In order to choose the most appropriate one among all the belief functions, we apply the principle of least commitment for evidential reasoning proposed by Dubois and Prade in [3], and Hsia in [4]

this principle can be considered as a natural approach for selecting the less specific *bba* from a subset , It consists in selecting the least committed *bba*, and supports the idea that one should never give more support than justified to any subset of Ω.

Among all the belief functions, the consonant, where focal elements are nested (there is a use of a relation of total ordering). The consonant are considered as the most appropriate functions because they express the principle of least commitment.
We are applying this principle to the consonant *bbd* in our illustration in section 6, to represent a normal distribution that is having a bell shape.

5 A distance between continuous belief functions

Traditional distances known for the discrete case are totally useless due to the nature of the continuous belief functions like described in section4. First of all, we have shown in section 3.2, a scalar product is able to compute a distance between two *bbd*. To handle the problem of the nature of these functions, a scalar product is defined on $\overline{\mathbb{R}}$ by:

$$\langle f,g\rangle = \int_{x=-\infty}^{+\infty}\int_{y=-\infty}^{+\infty} f([x,y])g([x,y])dxdy \tag{15}$$

In this section, we will introduce a new method to evaluate the similarity based on Jousselme's distance with Smets' formalism on continuous belief functions. A similarity /dissimilarity measure quantifies how much two distributions are different. Using the properties of belief functions on real numbers, we are now able to define a distance between two densities in a interval I.

$$\langle f_1,f_2\rangle = \tag{16}$$
$$\int_{-\infty}^{+\infty}\int_{y_i=x_i}^{+\infty}\int_{-\infty}^{+\infty}\int_{y_j=x_j}^{y_j=+\infty} f_1(x_i,y_i)f_2(x_j,y_j)\delta(x_i,x_j,y_i,y_j)dy_jdx_jdy_idx_i$$

The scalar product of the two continuous *pdfs* is noted: $\langle f_1,f_2\rangle$ satisfying all proprieties stated in section 3.2, with a function δ defined as $\delta:\mathbb{R}\longrightarrow[0,1]$

$$\delta(x_i,x_j,y_i,y_j) = \frac{\lambda([\![max(x_i,x_j),min(y_i,y_j)]\!])}{\lambda([\![max(y_i,y_j),min(x_i,x_j)]\!])} \tag{17}$$

where λ represents the Lebesgue measure used for the interval's length,

$\delta(x_i,x_j,y_i,y_j)$ is an extension of the measure of Jaccard applied for the intervals in the case of continuous belief functions where $[\![a,b]\!]$ refers to (17).

$$[\![a,b]\!] = \begin{cases} \emptyset, & \text{if } a>b \\ \text{[a,b]}, & \text{otherwise.}\end{cases} \tag{18}$$

Therefore, the distance is defined by:

$$d(f_1,f_2) = \sqrt{\frac{1}{2}(\|f_1\|^2+\|f_2\|^2-2\langle f_1,f_2\rangle)} \tag{19}$$

This distance can be used between two or more belief functions.

Let's consider σf a set of *bbds*. We measure the distance between one *bbd* and the $n-1$ other one by:

$$d(f_i,\sigma f) = \frac{1}{n-1}\sum_{j=1,i\neq j}^{n} d(f_i,f_j) \tag{20}$$

6 Illustrations

As mentioned in the previous section, the result of our work is a distance between two or several *bbds*.

In this section we consider the cases of two different kinds of distributions: the first one is a normal representation and the second is an exponential one. We use these distributions to deduce the different *bbds* and then we are able to measure the distance between two or several continuous belief functions.

The aim here is to have a probability distribution that includes uncertainty, so we model it using the basic belief densities.

6.1 Basic belief densities induced by normal distributions

In this analysis we will focus on a normal probability density function, like presented [9]. The focal sets of the belief functions are the intervals $[\mu - x, \mu + x]$ of $\overline{\mathbb{R}}$, with μ: the mean of the normal distribution and $x \in \mathbb{R}^+$ and σ: the standard deviation. We consider a normal distribution $\mathcal{N}(x; \mu; \sigma)$, with $x \geq \mu$:

$$\varphi(x) = 2(x-\mu)^2 \frac{1}{\sigma\sqrt{2\pi^3}} e^{\frac{(x-\mu)^2}{2\sigma}}, \tag{21}$$

where $\varphi(x)$ is the basic belief density associated to the Gaussian, when we apply the principle of least commitment where x is the representation of the intervals previously mentioned.

This function is null at $x = \mu$, increases with x and reaches a maximum of $4/(\sigma e\sqrt{2\pi})$ at $x = \mu + \sqrt{2\sigma}$, then decreases to 0 at x goes to infinity like presented by Smets in [15].

6.2 Analysis of a distance between belief densities induced by normal distributions

In this part, we will make a comparison between two distances one is measured using a classical scalar product and the other is our adaptation of Jousselme's distance for two and several continuous belief functions.

In Figure 1, we consider four *pdfs* having a normal distribution.

pdf	1	2	3	4
μ	0	0	4	4
σ	1	0.5	1	0.5

Table 1: Probability density distributions

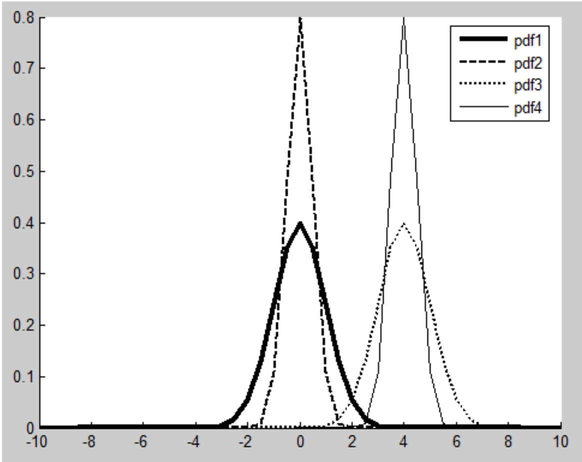

Fig. 1: Four *pdfs* following normal distribution.

First of all we measure the distance between f_1 and the rest of the *pdfs* using (19). After that measure the average of the distance between all the *bbds* according to (20).

The average of the distances is $d(f_1, \sigma f) = 0.634$.
According to the Figure 1, the smaller is the distance, the more similar are the distributions. The distance between f_1 and f_4 is the biggest one, this means that those two distributions are the farest from each other comparing to the distance between f_1, f_2, and f_1, f_3.
The distance between f_1 and f_2 is the smaller one, this can be explained in Figure 1 by the fact that f_1 is the nearest one tho f_2.

distance	value
$d(f_1, f_2)$	0.3873
$d(f_1, f_3)$	0.7897
$d(f_1, f_4)$	0.8247

Table 2: Distances measured.

In Figure 2, we fix $\mu_1 = 0$, $\sigma_1 = 0.5$, and for the second *pdf*, $0 \leq \mu_2 \leq 10$ with a step $0,5$ and $0.1 \leq \sigma_1 \leq 3$ with a step =0.1. For the normal distribution, we only use focal elements where $y = 2\mu - x$ if $\mu_1 \neq \mu_2$, then the distance based on the classical scalar product is null. Else, $y = 2\mu = x$, so the scalar product presented in the continuous domain is:

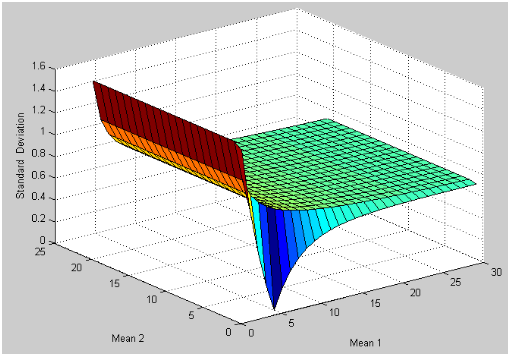

Fig. 2: Distance using a classical scalar product.

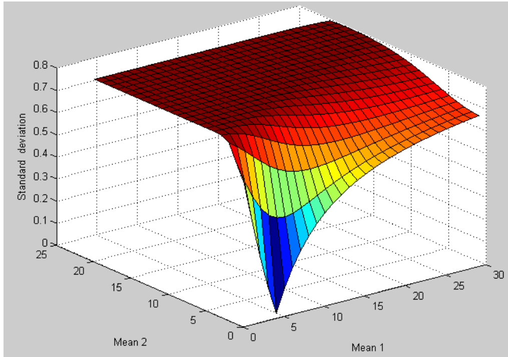

Fig. 3: Distance using Jaccard based on scalar product.

$$\int_{y=\mu}^{+\infty} f(y)g(y)dxdy. \tag{22}$$

After that, we consider the distance presented in (19) for continuous belief functions.

Classical Scalar Product for belief densities induced by normal distribution: In Figure 2, we notice that there is a remarkable drop in the value of the distance followed by a discontinuity in the 3D representation. When $\mu_1 = \mu_2$, the distance based on classical scalar product is not null, we can say that the standard deviation has an impact on the distance. Moreover, when σ_2 decreases, in this case the distance based on the classical scalar product has a rising values. However, reaching a certain point, especially when $\sigma_1 = \sigma_2$, we observe that the distance is null, so here we are in presence of similar distributions.

Moreover, the distance based on classical scalar product does not have a normalized value $d > 1,4$. Unfortunately, the mean does not really have an outstanding impact on the distance using the classical scalar product as a measure. Based on that, the distance based on classical scalar product is almost useless in this case, which can be considered as a bad representation of the distance. All these elements created a trays' phenomenon in the obtained Figure 2.

Continuous distance for belief densities induced by normal distribution: By varying the values of μ_2 between $[0, 10]$ with a step of 0.5 we obtain Figure 3. When $\mu_1 = \mu_2$, and $\sigma_1 = \sigma_2$, we are dealing with similar distributions, we obtain a null distance. Otherwise, wa also notice that when the difference

between σ_1 and σ_2 increases, the distance rises. We have a similar behavior also when the difference between the means μ_1 and μ_2, every time, the similarity between the two distributions drops which is explained by a growth of the value of the distance.
Comparing to the results obtained with the classical scalar product, we observe that our distance takes in consideration the standard deviation, that does have a real impact when computing this measure of similarity.

The difference between them, is the function δ presented in (17). δ, that allows us to have a more specific distance between two belief functions and use wisely the mean to have a better measure of the distance.

It is more useful, then and accurate to use our distance proposed in (19).

6.3 Basic belief densities induced by exponential distributions

In this section, we will suppose that the probability distribution is following an exponential density. The specific expression is for the probability density.

$$f(y) = \frac{y}{\theta^2} e^{\frac{-y}{\theta}} \tag{23}$$

It is obtained when we use the Least Commitment Principle presented in Section 4.2, on a set of basic belief densities associated to the exponential distribution, where θ is the mean and the focal elements are in the intervals $[0, x]$.

6.4 Analysis of a distance between belief densities induced by exponential distributions

We measure the distance between 2 exponential distributions according to a classical scalar product definition, and our adaptation of Jousselme's distance for continuous belief functions.

Figures 4 and 5 show respectively the results obtained after computing the two distances. We are dealing with two exponential distributions f_1, f_2, where $\theta_1 = 1$ and $\theta_2 \in [0.1, 10]$ with a discretization step $= 0.1$.

Classical Scalar Product for belief densities induced by exponential distributions: At the begging of Figure 4, when $\theta_1 = 1$ and $\theta_2 = 0.1$, the distance based on classical scalar product has the highest value. When the value of θ_2 increases, the distance between the distributions decreases continually until it makes a discontinuity when $\theta_1 = \theta_2$ where it has a null value, this means that $f_1 = f_2$.

After that, the value of θ_2 gets far from θ_1, the probability distribution f_2 becomes different from f_1 and that generates a non null distance that increases suddenly just when θ_2 gets bigger, then the distance based on classical scalar product decreases and gets stable and reaches a value near to 0.4. Based on these observations, the distance using the classical scalar product is not really adapted

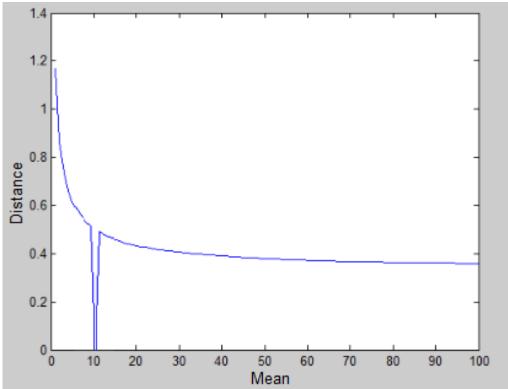

Fig. 4: Distance using a classical scalar.

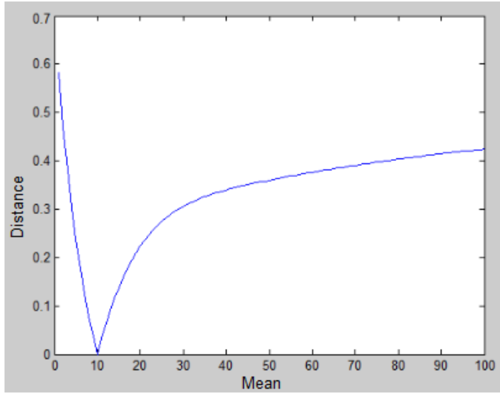

Fig. 5: Distance using Jaccard based scalar product.

for continuous belief functions because it creates a discontinuity, and except the point where $\theta_1 = \theta_2$, the variation of this distance does not reflect the difference between θ_1 and θ_2.

Continuous distance for belief densities induced by exponential distributions: The behavior of the distance in Figure 5 is starting at the highest value $d(f_1, f_2) = 0.6$ with the similar exponential distribution presented previously. When the value of θ_2 gets closer to θ_1, $d(f_1, f_2)$ decreases and reaches a null value where $f_1 = f_2$ (the distributions are similar to each other). Unlike the the distance based on classical scalar product, the increase of $d(f_1, f_2)$ is gradual as θ_2 gets bigger values, the distance grows continually.

7 Conclusion

In this paper we have introduced an adaptation of the Jousselme's distance for the continuous belief functions according to Smets' formalism. This measure is able to define a distance between two or several basic belief density functions.

This distance is based on the function δ, which take into account the imprecision of the focal elements, whereas the classical scalar product. This distance has all the properties of a classical distance.

To illustrate the behavior of the proposed distance in different situations, we used different probability distributions: a normal and an exponential one, from which we deduced basic belief densities. Afterwards, we compared our distance to the results obtained when using a distance based on a classical scalar product.

In future work, we will use the proposed distance in order to define a conflict measure for continuous belief functions.

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